Evaluation of Angular Distribution Models to Estimate Sky Diffuse Irradiance on Tilted Planes in Urban Environments

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Abstract

The great growth of cities in the last century, together with the scarcity of developable land, has led to the growth of high-rise cities, limiting the access to solar radiation for their inhabitants. For this reason, it is crucial to have models that allow an accurate assessment of the amount of solar radiation that can be received in highly obstructed environments. In this work, it has been evaluated and compared the performance of an angular distribution model against a simpler irradiance model to estimate the diffuse irradiance received on tilted planes located in obstructed environments. To this end, a total of 320 urban scenarios were considered derived from the combination of different orientations and inclinations of the tilted plane, and different orientations and aspect ratios of the urban canyon. Models were evaluated against the diffuse irradiance values obtained by the ISO 15469:2004(E)/CIE S 011/E:2003 model and radiance measurements performed by a sky scanner at the radiometric station of the Public University of Navarre (Spain).

Keywords: Diffuse solar irradiance, Angular distribution models, Tilted planes, Urban environments

1. Introduction

For the last century there has been a growing concentration of population in cities. This tendency is expected to continue unless the necessary policy measures are taken. According to the United Nations report (United Nations, 2018), in 2018, 1.7 billion people –23 per cent of the world's population– lived in a city with at least 1 million inhabitants. In 2030, a projected 28 per cent of people worldwide will be concentrated in cities with at least 1 million inhabitants. The scarcity of available developable land has caused cities to grow in height, limiting access to solar radiation for their inhabitants. For this reason, it is crucial to have models that allow an accurate assessment of the amount of solar radiation that can be received in highly obstructed environments. Specifically, the estimation of diffuse irradiance on tilted planes –given their diffuse and anisotropic nature– requires the use of models. Depending on their complexity, such models can be classified into two main groups: (1) simple or irradiance models within which it is possible to distinguish between isotropic and pseudo-isotropic models (Liu and Jordan, 1961; Koronakis, 1986; Tian et al., 2001) and anisotropic models (Bugler, 1977; Temps and Coulson, 1977; Klucher, 1979; Steven and Unsworth, 1979; Hay and Davies, 1980; Willmott, 1982; Ma and Iqbal, 1983; Skartveit and Olseth, 1986; Saluja and Muneer, 1987; Gueymard, 1986, 1987; Muneer, 1987, 1990; Perez et al., 1987, 1990; Reindl et al., 1990); and (2) complex or angular distribution models (Matsuura and Iwata, 1990; Perez et al., 1993a; Brunger and Hooper, 1993; Igawa et al., 2004; Igawa, 2014; Lou et al., 2022).

Although numerous studies can be found in the scientific literature that evaluate the performance of these models, both simple and complex, when estimating the irradiance received on an inclined plane located in an obstacle-free environment, there are very few studies that evaluate this performance in obstructed environments. In this sense, it was carried out an evaluation of the behavior of simple or irradiance models to assess diffuse irradiance on inclined planes in urban environments in a previous work (García et al., 2021).

The current work aims to evaluate and compare the performance of an angular distribution model against a simple

doi:10.18086/eurosun.2022.11.06 Available at http://proceedings.ises.org

irradiance model, both widely used, in determining sky diffuse irradiance in obstructed environments. The hypothesis is that models based on the angular distribution of diffuse radiance in the sky allow for more accurate consideration of environmental obstacles than the simple models. The scope of this study is limited to sky diffuse irradiance. That is, we have not considered the reflected fluxes which, in very obstructed environments, can account for a significant part of the received energy. Therefore, this issue will be addressed with the depth it deserves in future works. The estimation of direct irradiance has also not been addressed.

This paper is organized into five sections and two appendices. The meteorological data and the quality control procedures are detailed in Section 2. Section 3 describes the general methodology and the different considered models. Section 4 presents the results obtained and the conclusions are detailed in Section 5.

2. Meteorological data

In order to compare the results obtained in this study with those obtained in the previous one (García et al., 2021), in which irradiance or simple models were evaluated, the same experimental dataset was used. Thus, the data used in this study were collected from the radiometric station of the UPNA, located on the roof of one of the buildings of the Higher Technical School of Agricultural Engineering and Biosciences (42º47'32'' N, 1º37'45'' W, 435 m a.s.l.) in Pamplona (Spain). The data series consist in 6,767 observations made between July and December 2018. Measured variables include 1-min frequency data of global irradiance on the horizontal plane, using a Kipp & Zonen CM11 pyranometer, diffuse irradiance on the horizontal plane with a Kipp & Zonen CM11 pyranometer with a shadow ball, and direct normal irradiance with a Kipp & Zonen CH1 pyrheliometer. All three instruments were mounted on a Kipp & Zonen 2AP solar tracker. In addition, the angular distribution of diffuse sky radiance was measured every 10 min using an EKO MS-321LR sky scanner. Both irradiance and angular distribution measurements corresponding to solar elevations below 5º were discarded. Irradiance measurements were further subjected to the quality control procedure proposed by the MESoR project (Hoyer-Klick et al., 2008). In the case of angular distribution of radiance, quality control was based in three criteria described in García et al. (2020).

3. Methodology

The general methodology consists of: (1) the sky classification according to the 15 types established by the ISO 15469:2004(E)/CIE S 011/E:2003 standard (2004) from radiance measurements performed by the sky scanner; (2) the estimation of the diffuse irradiance on the tilted plane from the measured angular distribution of radiance and the ISO/CIE angular distribution model, considering the obstruction provided by the urban canyon; (3) obtaining the diffuse irradiance predicted by the angular distribution model and the irradiance one, considering the effect of obstructions; and (4) the comparison of model predictions with the reference irradiance values obtained in step 2 by the ISO/CIE angular distribution model.

The computation of the diffuse irradiance received on the tilted plane of interest, indicated in steps 2 and 3 of the general methodology, was carried out by integrating the radiances corresponding to a series of visible sky elements –782,268 in this work– on such plane. In this way, those sky elements obstructed by the urban environment can be accurately discarded (see Fig. 1).

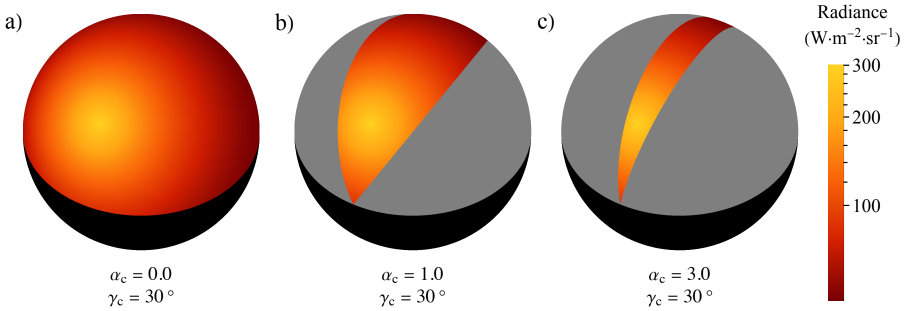

Fig. 1: Projection of the ISO/CIE sky standard 7 onto a plane with an inclination β_p = 45º and an azimuth γ_p = 30º, when the sun has a solar zenith angle of 45º and an azimuth of 30º, considering different aspect ratios (α_c) of the urban canyon. The black-shaded area is the visible part of the ground, and the gray-shaded area is the projection of the urban canyon onto the tilted plane.

The described methodology was used to compare the performance of an angular sky diffuse radiance distribution model versus an irradiance model when estimating the diffuse irradiance received on tilted planes located in urban environments with different configurations. The considered models –listed below–, as well as the reference ISO/CIE angular distribution model, are described in Subsections 3.1 to 3.3.

1. All-weather model (Perez et al., 1993a, 1993b). This is a widely used angular distribution model that uses the sky's clearness (ε) and brightness (Δ) to parameterize the angular distribution of luminance in the sky. Despite being designed to estimate the angular distribution of sky luminance, previous studies have confirmed its good performance when estimating radiance (Gracia et al., 2011).
2. Modified Perez irradiance model (Perez et al., 1987). This is a modification of the original anisotropic sky model proposed by García et al. (2021) to take into account the effect of the obstacles on the various defined sky areas (isotropic background and circumsolar region). As in the case of the All-weather model, ε and Δ are used to characterize the sky conditions.

As in the case of the data set used, a similar set of urban scenarios of the previous work was considered in order to compare the results obtained previously with simple models, and those obtained in this study from a complex one. Therefore, the general methodology was applied to a total of 320 urban scenarios resulting from the combination of 4 different orientations (γ_p) and 10 inclinations (β_p) of the tilted plane and 2 orientations (γ_c) and 4 aspect ratios (α_c) of the urban canyon, listed in Tab. 1. The aspect ratio is considered as the quotient of the height and width of the urban canyon and the azimuth origin is in the south and it increase positively towards the west and negatively towards the east.

Many urban scenarios were considered to cover as many situations as possible that can occur in a real urban environment. For all scenarios, the tilted plane of interest was considered to be in the center of the urban canyon, i.e., equidistant to the two facades that delimit the canyon.

Tab. 1: Considered configurations of the urban canyon and tilted plane.

Variable	Initial value	Final value	Increment	Cases
Urban canyon azimuth (γ_c)	0º	90º	90º	2
Urban canyon aspect ratio (α_c)	1	4	1	4
Plane tilt angle (β_p)	0º	90º	10º	10
Plane azimuth (γ_p)	-180º	90º	90º	4

The ISO/CIE angular distribution model was used to obtain the reference diffuse irradiance against which values provided by the analyzed irradiance models were compared because of two fundamental reasons. The first is the practical impossibility to install irradiance sensors in such a large number of considered scenarios. The second is to overcome data discontinuities resulting from the saturation of the sky scanner measurements of angular distribution of radiance in those patches close to the sun. The measurements of angular distribution of sky radiance were used to characterize the state of the sky according to the ISO/CIE standard. In this way, it was possible to determine the theoretical ISO/CIE radiance distribution closest to the real measured distribution. Then, the model proposed in the standard was used to obtain a continuous distribution of radiance in the sky according to the obtained ISO/CIE sky-type.

3.1. Reference luminance/radiance model: ISO/CIE angular distribution model

The standard ISO 15469:2004(E)/CIE S 011/E:2003 (2004) established a total of 15 standard sky luminance distributions. Although the ISO/CIE model is designed to describe the luminance distribution of the sky and not the radiance, García et al. (2020) found that classification in sky types from radiance and luminance measurements gives an exact match for nearly 60% of the cases and reaches 90% when, in addition to the matching classifications, the differences of 1 and 2 sky types are considered.

According to the ISO/CIE standard, the luminance relative to zenith at a given point in the sky vault, $l_{CIE}(\theta,\chi)$, is given by eq. (1).

$$l_{CIE}(\theta,\chi) = \frac{L_{CIE}(\theta,\chi)}{L_Z} = \frac{\varphi(\theta)\, f(\chi)}{\varphi(0)\, f(\theta_z)}, \qquad \text{(eq. 1)}$$

where $L_{CIE}(\theta,\chi)$ is the luminance of any sky point given its zenith angle (θ) and angular distance to the sun (χ), $f(\chi)$ is the indicatrix function, $\varphi(\theta)$ is the gradation function, θ_z is the solar zenith angle.

The gradation function, $\varphi(\theta)$, which obeys to the eq. (2), characterizes the luminance variation from the zenith ($\theta = 0°$) to the horizon ($\theta = 90°$).

$$\varphi(\theta) = 1 + a_{CIE}\exp(b_{CIE}/\cos\theta). \qquad \text{(eq. 2)}$$

The indicatrix function, $f(\chi)$, in eq. (3) expresses the relationship between the luminance at a sky point and that at the point where $\chi = 90°$). The indicatrix function is related to the scattering of solar radiation as it passes through the atmosphere.

$$f(\chi) = 1 + c_{CIE}\left[\exp(d_{CIE}\chi) - \exp(d_{CIE}\,\pi/2)\right] + e_{CIE}\cos^2\chi. \qquad \text{(eq. 3)}$$

Coefficients a_{CIE}, b_{CIE}, c_{CIE}, d_{CIE} and e_{CIE} included in eqs. (2) and (3) depend on the standard sky type. Consequently, in order to calculate the angular distribution of relative luminance in the sky vault at any given moment by applying eq. (1), it is necessary to know first the type of sky at that moment.

When L_Z is not a known value, it is possible to obtain the absolute luminance of the sky element, $L_{CIE}(\theta,\chi)$, from the normalization of the measured horizontal diffuse irradiance (G_d) according to eq. (4).

$$L_{CIE}(\theta,\chi) = \frac{l_{CIE}(\theta,\chi)\,G_d}{\int_{\theta=0}^{\pi/2}\int_{\gamma=0}^{2\pi}[l_{CIE}(\theta,\chi,\gamma)\cos\theta]\mathrm{d}\theta\,\mathrm{d}\gamma}. \qquad \text{(eq. 4)}$$

Considering the proposed sky discretization, which consists of a large number of sky elements –782,268 in this work–, the integration of the denominator of eq. (4) can be replaced by a sum without loss of precision, as described in eq. (5).

$$L_{CIE}(\theta,\chi) = \frac{l_{CIE}(\theta,\chi)\,G_d}{\sum_{i=1}^{n} l_{CIE,i}(\theta,\chi)\omega_i\cos\theta_i}, \qquad \text{(eq. 5)}$$

where n is the number of sky elements into which the sky vault is divided and ω_i is the solid angle of the sky element i.

So that, according to the ISO/CIE sky radiance distribution, the horizontal diffuse irradiance ($G_{d,CIE}$) can be obtained by integrating/summing the values of $L_{CIE}(\theta,\chi)$, according to eq. (6).

$$G_{d,CIE} = \sum_{i=1}^{n} L_{CIE,i}(\theta,\chi)\omega_i\cos\theta_i. \qquad \text{(eq. 6)}$$

This last operation, as it is, may be of no interest since the value of G_d is already known. However, when calculating the diffuse irradiance received on a tilted plane ($G_{d,T,CIE}$) located in an obstructed or unobstructed environment, an analogous procedure will be applied, eq. (7).

$$G_{d,T,CIE} = \sum_{i=1}^{n} max\left[L_{CIE,i}(\theta,\chi)\omega_i\cos\theta_i', 0\right], \qquad \text{(eq. 7)}$$

where θ_i' is the zenith angle relative to the normal of the tilted plane of a sky element i.

3.2. Luminance/radiance model: All-weather model

According to Perez et al. (1993a, 1993b), the relative luminance, $l_{Aw}(\theta,\chi)$, defined as the ratio between the luminance of the considered sky element, $L_{Aw}(\theta,\chi)$, and the luminance of an arbitrary sky element –generally the zenith– is given by eq. (8).

$$l_{Aw}(\theta,\chi) = [1 + a_{Aw}\exp(b_{Aw}/\cos\theta)][1 + c_{Aw}\exp(d_{Aw}\chi) + e_{Aw}\cos^2\chi]. \qquad \text{(eq. 8)}$$

The coefficients a_{Aw}, b_{Aw}, c_{Aw}, d_{Aw} and e_{Aw} are adjustable coefficients that depend on two parameters used to characterize the sky conditions at a given time: sky clearness (ε) and sky brightness (Δ).

$L_{Aw}(\theta,\chi)$ may be obtained from $l_{Aw}(\theta,\chi)$, if zenith luminance (L_z) is known by eq. (9).

$$L_{Aw}(\theta,\chi) = \frac{L_{z,Aw}\,l_{Aw}(\theta,\chi)}{l_{Aw}(0,\theta_z)}. \qquad \text{(eq. 9)}$$

However, the authors recommend that $L_{Aw}(\theta,\chi)$ be obtained by eq. (10) after normalization of G_d. According to the reasoning stated in Section 3.1, the integration can be replaced by a sum of very small discrete elements.

$$L_{Aw}(\theta,\chi)=\frac{l_{Aw}(\theta,\chi)\,G_d}{\int_{\theta=0}^{\pi/2}\int_{\gamma=0}^{2\pi}[l_{Aw}(\theta,\chi)\cos\theta]\mathrm{d}\theta\,\mathrm{d}\gamma}=\frac{l_{Aw}(\theta,\chi)\,G_d}{\sum_{i=1}^{n}l_{Aw,i}(\theta,\chi)\omega_i\cos\theta_i}. \qquad \text{(eq. 10)}$$

Once the value of $L_{Aw}(\theta,\chi)$ is known, the calculation of the diffuse irradiance on a horizontal ($G_{d,Aw}$) or tilted plane, ($G_{d,T,Aw}$) can be carried out by eqs. (6) and (7), respectively.

3.3 Irradiance model: modified Perez model

The original model published by Perez et al. (1987) is shown in eq. (11). The three terms in this equation correspond to the diffuse irradiance from the isotropic background region, the circumsolar region, and the contribution of the horizon brightness.

$$G_{d,T,Perez}=G_d\left[(1-F_1)\left(\frac{1+\cos\beta_p}{2}\right)+F_1\frac{a}{b}+F_2\sin\beta_p\right], \qquad \text{(eq. 11)}$$

where F_1 is the circumsolar brightness coefficient; F_2 is the horizon brightness coefficient; a is the solid angle subtended by the circumsolar region, weighted by its average incidence on the slope; and b is the solid angle subtended by the circumsolar region, weighted by its average incidence on the horizontal. Coefficients F_1 and F_2 depend on ε and Δ.

The proposal by García et al. (2021) modified the original model in order to take into account the effect of urban canyon on the irradiance of the isotropic background and the circumsolar region. In this regard, the first term, the contribution of the isotropic background, was modified replacing the factor $(1+\cos\beta_p)/2$ by the sky view factor (SVF). In the case of the circumsolar component, the factor a/b was replaced by the circumsolar view factor (CVF), which allows to consider the effect of the urban canyon on the circumsolar region. The term related to the horizon brightness was eliminated, in the understanding that this sky region will be obstructed, almost completely, by the urban canyon. Therefore, the modified Perez model is given by Equation (12).

$$G_{d,T,Perez}=G_d[(1-F_1)SVF+F_1CVF]. \qquad \text{(eq. 12)}$$

García et al. (2021) analyzed 5 different half-angles of opening of the circumsolar region –from point source to 45º– and demonstrated that the model that considered the 35º half-angle circumsolar region performed best when estimating the irradiance received on a tilted plane considering all studied urban scenarios. For this reason, in this work such aperture angle of the circumsolar region was selected.

4. Results and discussion

Fig. 2 shows the sky classification obtained according to the ISO/CIE standard. Clear sky types (11-15) predominate, with 56% of the records, being the sky type 13 the most frequent one. Overcast skies (types 1-5) accounted for 24.9% of the skies observed and intermediate skies (types 6-10) 19.1%.

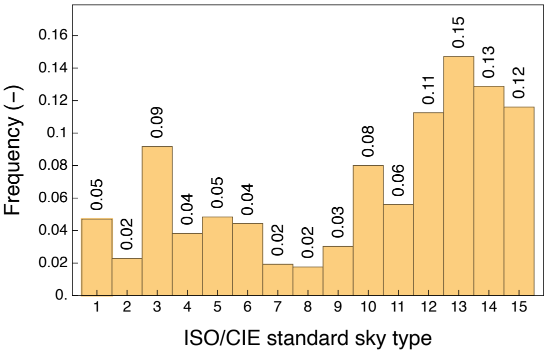

Fig. 2: Observed frequency of occurrence of each ISO/CIE Standard General Sky classification.

Once the ISO/CIE sky type was obtained, the diffuse irradiance on the inclined plane was determined for each of the 320 urban scenarios designed using the ISO/CIE angular distribution model. These irradiance values were used as the reference against which the irradiance estimates provided by the All-weather angular distribution model, or the modified Perez irradiance model were compared. For each scenario and modified model, the performance was evaluated with the relative mean square error ($rRMSE$), according to eq. (13).

$$rRMSE(\%) = \frac{100}{\bar{G}_{d,T,CIE}} \sqrt{\frac{\sum_{i=1}^{n} \left(G_{d,T,i,CIE} - G_{d,T,i,x}\right)^2}{n}}. \qquad \text{(eq. 13)}$$

where $G_{d,T,CIE}$ is the diffuse irradiance on the tilted plane calculated from the ISO/CIE angular radiance distribution model, $\bar{G}_{d,T,CIE}$ is the average of all $G_{d,T,CIE}$ values, and $G_{d,T,i,x}$ is the diffuse irradiance on the tilted plane given by the All-weather angular distribution model or the modified Perez irradiance model.

Fig. 3 shows the $rRMSE$ values obtained by the two evaluated models considering a γ_c of 0º (north-south direction) and a γ_p of -90º (east-oriented plane), Fig. 3a, and of 0º (south-oriented plane), Fig. 3b. In each of the plots we show the evolution of the errors as a function of the plane inclination and the aspect ratio of the canyon (α_c). Although 4 different orientations of the plane were considered for γ_c = 0º, only 2 have been included in Fig. 3 for space reasons. In all cases it can be observed how the error obtained by the models when estimating the diffuse irradiance on a tilted plane increases as the plane's inclination and the aspect ratio of the urban canyon rise. However, the $rRMSE$ values are significantly lower in the case of the angular distribution model compared to the irradiance model.

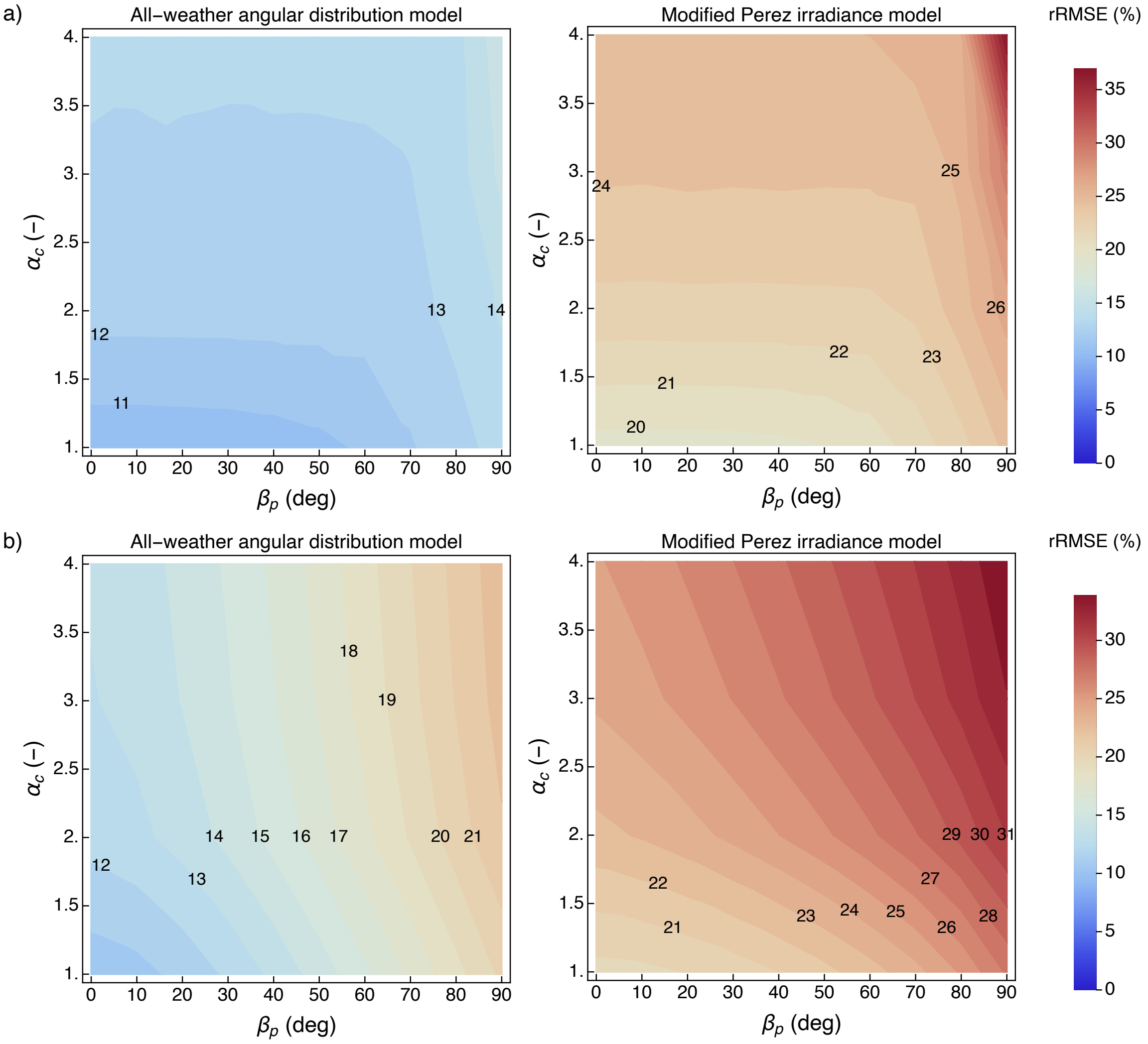

Fig. 3: $rRMSE$ (%) values of tilted diffuse irradiance obtained by each model for the different β_p and α_c combinations considering a γ_c of 0º (north-south direction) and a γ_p of (a) -90º (east facing) and (b) 0º (south facing).

Fig. 4 shows results analogous to those presented in Fig. 3, but, in this case, for a γ_c of 90º (east-west direction). Again, the trend observed in Fig. 3 can be seen. That is, the errors increase with the increase of the β_p and α_c, being significantly higher in the case of the simple model or irradiance model. For the specific case of this canyon orientation and for the reasons mentioned above, the other 2 plots corresponding to plane orientations of 90º and 180º have not been included. Thus, a total of 8 plots corresponding to the 320 scenarios considered were obtained.

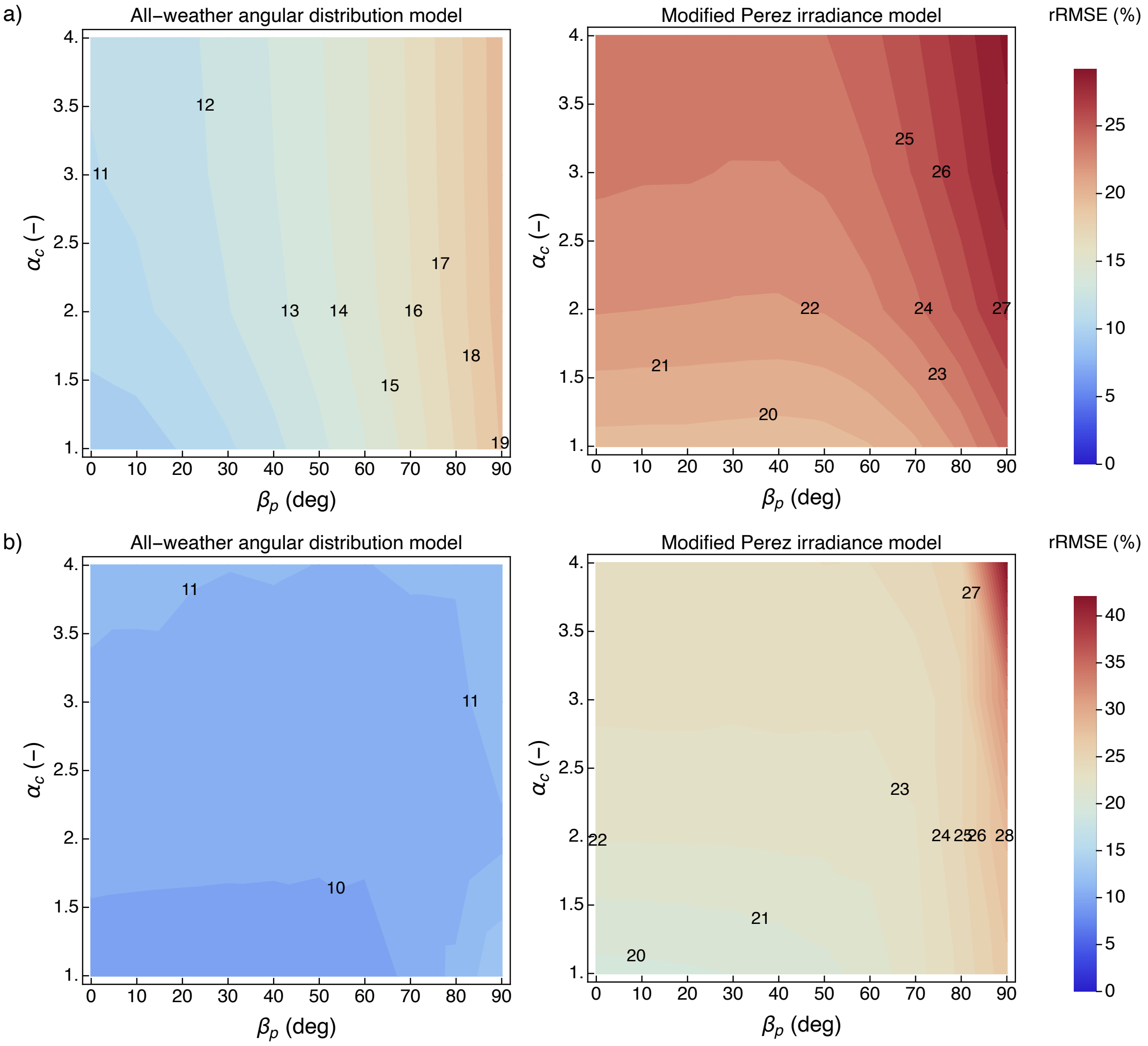

Fig. 4: $rRMSE$ (%) values of tilted diffuse irradiance obtained by each model for the different β_p and α_c combinations considering a γ_c of 90º (east-west direction) and a γ_p of (a) -90º (east facing) and (b) 0º (south facing).

Data in those 8 plots like Figs. 3 and 4 were summarized to simplify the analysis of the results. In this regard, Tabs. 2 and 3 show the $rRMSE$ values obtained for the two considered γ_c values. Each table shows the errors obtained by the two analyzed models for the different α_c. Also, for each α_c, models have been ranked according to their $rRMSE$ values and their rank order is shown in brackets.

As can be seen, in all cases the All-weather model, which considers the angular distribution of radiance in the sky vault, shows the best performance when estimating the $G_{d,T}$ values received on a plane located inside an urban canyon, with errors ranging from 11.96% in an obstacle-free environment to 15.23% when the aspect ratio of the urban canyon reaches a value of 4. In fact, the $rRMSE$ values obtained by the simple or irradiance model almost double the errors of the angular distribution model.

Tab. 2: $rRMSE$ (%) values of tilted diffuse irradiance obtained by each model for different α_c considering a γ_c of 0° (N-S). For each α_c, model rank scores are shown in parenthesis.

α_c	All-weather angular distribution model	Perez modified irradiance model
1	13.01 (1)	21.00 (2)
2	14.52 (1)	23.65 (2)
3	15.02 (1)	25.00 (2)
4	15.23 (1)	26.04 (2)

Tab. 3: $rRMSE$ (%) values of tilted diffuse irradiance obtained by each model for different α_c considering a γ_c of 90° (E-W). For each α_c, model rank scores are shown in parenthesis.

α_c	All-weather angular distribution model	Perez modified irradiance model
1	11.70 (1)	20.68 (2)
2	12.35 (1)	23.12 (2)
3	12.64 (1)	24.37 (2)
4	12.75 (1)	25.52 (2)

5. Conclusions

In this work we have evaluated the performance of an angular distribution model of diffuse sky radiance versus that of an anisotropic irradiance model, when estimating the diffuse irradiance received in a tilted plane immersed in an urban canyon. After analyzing the results considering a total of 320 urban scenarios, which arise from the combination of different orientations and aspect ratios of the canyon and different orientations and inclinations of the plane of interest, the starting hypothesis of the article has been verified. That is, the angular distribution model performs significantly better than the irradiance model. In fact, when comparing the models, it has been observed that the errors are reduced by almost half.

It should be noted that the values of sky diffuse tilted irradiance taken as a reference against which to assess the considered models have been calculated according to the ISO/CIE angular distribution model, given the practical impossibility of carrying out a measurement campaign in the 320 urban scenarios. Although this may introduce some uncertainty in the results, the use of this model, together with the sky scanner measurements, makes it possible to obtain the standard sky radiance distribution most similar to the real measured distribution.

In the future, this same study will be undertaken using HDR images of the sky calibrated in radiance, which will allow greater precision in the results. The problem of estimating reflected energy fluxes in complex environments will also be addressed.

In conclusion, the use of angular distribution models is recommended over irradiance models when estimating the diffuse sky irradiance received on planes located in urban environments.

6. Acknowledgments

The authors gratefully acknowledge the financial support provided by the Spanish Ministry of Science & Innovation under the I+D+i state program “Challenges Research Projects” (Ref. RTI2018-098900-B-I00). Ignacio García thanks to the Spanish Ministry of Universities and the European Union-Next Generation EU for their financial support (Program for the requalification of the Spanish university system 2021-2023, Resolution 1402/2021). Diego Granados-López thanks grant PIRTU Program, ORDEN EDU/556/2019, for its financial support.

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