

## Development of A Spectral Integral Method for Analyzing Solar Effects through Windows on Indoor Thermal Comfort

Nan Wang, Yanxiao Feng and Julian Wang

Department of Architectural Engineering, Pennsylvania State University, State College (USA)

Corresponding Author: Julian Wang, [julian.wang@psu.edu](mailto:julian.wang@psu.edu)

### Abstract

The interaction among the spectral characteristics of solar radiation, windows, and human skins may affect indoor thermal comfort. For analyzing such shortwave solar effects through windows on indoor thermal comfort, a more explicit method taking spectral features into account is indispensable. We built this new calculation methodology, called spectral integral method, upon a previous work that was mainly used to transfer the shortwave solar effect into the equivalent longwave effect to calculate indoor thermal comfort with the Predicted Mean Vote. Compared with the previous ones centered in the constant and simplified radiometric quantities, the uniqueness of this method is to provide the underlying radiometric calculation with the detailed and precise spectral contents and their variations of radiation sources, mediums, and receivers details. We adopted solar irradiance data from 8:00 to 18:00 with an interval of one hour in Denver, Colorado in a case study to verify the necessity of the spectral integral method relative to the constant method. The Predicted Mean Vote values with constant and spectral integral methods were then compared. The result of this work shows that using the spectral integral method could lead to quite different decisions of estimating indoor thermal comfort in some circumstances in terms of solar intensities and solar positions.

*Keywords: Solar radiation, Spectral integral method, Indoor thermal comfort, Mean radiant temperature, Building windows*

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## 1. Introduction

Human thermal comfort has drawn increasing attention under the escalating demand for a high-quality and healthy living environment. The Predicted Mean Vote (PMV) model in ASHARE 55 standard (ASHARE, 2010) has been widely adopted to predict indoor thermal comfort by using six parameters, including indoor air temperature, mean radiant temperature (MRT), air speed, relative humidity, metabolic rate, and clothing level. On the other side, solar radiation, which is indispensable in daily life, possesses huge energy and is an essential factor in affecting thermal comfort. In particular, solar radiation may exhibit a heating effect on indoor occupants not only through longwave radiation emitted from the heated air, envelopes, and surrounding objects but also via shortwave radiation transmitted through windows to body surfaces, as stated in (Peter, 2000). When predicting thermal comfort, the quantitative contribution of longwave radiation could be represented by the parameter of MRT in the PMV model. However, the shortwave effect could not be simply and directly represented by any parameters in the PMV model. An equivalent conversion is needed to convert the thermal effect of shortwave radiation to the longwave radiation by eq. 1 (Edward, 2015). The quantitative contribution of the additional longwave radiation that is converted from the shortwave radiation is called mean radiant temperature delta (MRT delta). By adding the MRT delta to the previous longwave-based MRT, the thermal effect of the shortwave radiation could be involved in the PMV model.

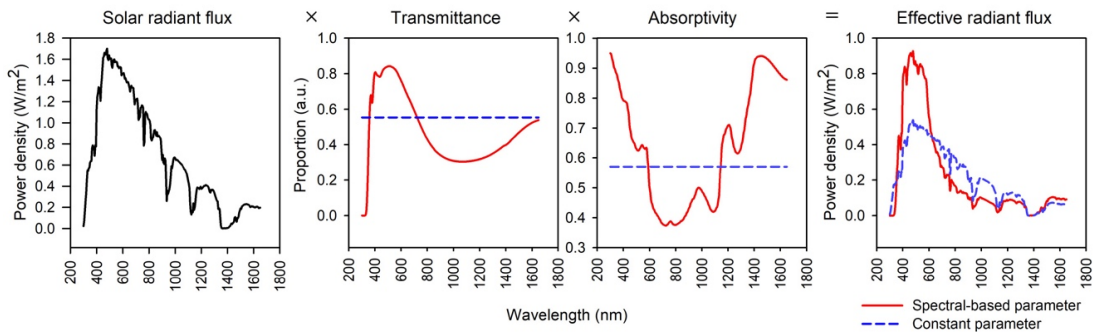
$$\alpha_{LW}ERF_{solar} = \alpha_{SW}E_{solar} \quad (\text{eq. 1})$$

Where,  $E_{solar}$  is the shortwave solar radiant flux on human skin;  $\alpha_{SW}$  is the shortwave absorptivity of human skin;  $ERF_{solar}$  is the transformed longwave radiant flux on human skin;  $\alpha_{LW}$  is the longwave absorptivity of human skin.

Before converting the shortwave radiation to the equivalent longwave radiation, it is important to know the

amount of shortwave radiation that works on human thermal perception. There are multiple energy nodes in the energy transfer process from the solar to the human perception. Knowing the nodes is helpful to understand the parameters involved in such transfer processes. The first node is the amount of solar radiation that arrives at exterior building windows, which is determined by the irradiance emitted from the solar and the percentage of the emitted solar irradiance in the orientation of the windows. Afterward, it is the second node representing the amount of the shortwave radiation transmitted through the windows, which mainly depends on the spectral characteristics of the windows in use. Next, the third node is the amount of the incident shortwave radiation on occupant body surfaces, in which the position, posture, exposed skin surface areas, and the sky view factor of the occupant play important roles. The final node is the amount of the shortwave radiation absorbed by the occupant, being affected by the spectral absorptivity of the human skin.

In the context of the solar effects on indoor thermal comfort, the most representative work is done by Edward et al (2018). They have formed a program called SolaCal, which may quantify the effect by using eight parameters and derive two primary variables:  $ERF_{solar}$  and MRT delta. As such, the thermal comfort PMV indicators could be estimated. Those eight parameters include the posture of an occupant, solar altitude ( $\beta$ ), solar horizontal angle relative to the front of a person (SHARP), direct beam (normal) solar irradiance ( $I_{dir}$ ), total solar transmittance ( $T_{sol}$ ), sky vault view fraction (fsvv), fraction of body exposed to the sun (fbes), average shortwave absorptivity ( $\alpha$ ). However, it is worth mentioning that among these parameters the total solar transmissivity and average shortwave absorptivity are the average optical properties based on the broadband, which ignores the spectral characteristics and the interactions of the radiation source (solar), medium (window), and the receiver (occupant). Notably, solar radiation has a spectral integral distribution, which means the photon carries different power at different wavelengths. Meanwhile, in the spectral range of solar radiation, building windows may have significantly different spectral features. For instance, as demonstrated in our previous studies (Julian, 2017), two spectrally selective glazing systems with very different spectral distributions may achieve very similar average transmittance under the standard solar light. Similarly, our body skins also have wavelength-dependent optical responses across different races and geographic regions. The superposition of the spectral features of solar radiation, window transmissivity, and human skin absorptivity can further complicate the resulting values. Figure 1 depicts the difference of superposition by wavelengths and by constant values, which uses constant spectra to represent constant values for comparing. Thus, in short, it is necessary to explore the influence of considering spectral characteristics of the parameters in predicting thermal comfort.



**Fig. 1: Spectral difference of effective radiant flux between multiplying spectral integral and constant solar transmissivity of window and absorptivity of human skin**

In this paper, we propose a spectral integral method to predict the solar effect on indoor thermal comfort, and we will also validate the necessity of the spectral integral method relative to the approach with corresponding constant values at different wavelengths (or called constant method in the following sections). The factors that influence the differences between the two methods are also explored. The unique contribution of this effort is to bring a more accurate analytical approach based on radiometry into the investigation of thermo-optical interactions among solar, glazing, and human.

## 2. Methods

### 2.1. Spectral integral method to predict human thermal comfort

The calculation for predicting human thermal comfort is by the PMV model. Among the six parameters in the PMV model, MRT could be related to the solar effect by adding the additional MRT delta term that converts the shortwave solar radiation to equivalent longwave radiation. The calculation to get the MRT delta value has been illustrated in the study mentioned above about SolarCal. Differently, for the two spectral integral parameters among eight predicting parameters in SolarCal, a spectral integral calculation in this work was proposed and then used instead of using provided or suggested constant values. We computed the spectral transmitted solar radiation that transmitted through the window by multiplying the spectral transmissivities of windows and the spectrum of the solar radiation by each wavelength and then summing the product at each wavelength together across all the wavelengths. The spectral integral solar transmissivity of the window ( $T_{sol_s}$ ) would then be the spectral transmitted solar radiation that transmitted through the window over the incident solar radiation. In a parallel fashion, the final solar radiation absorbed by occupants was obtained through multiplying the above three factors: solar spectra, window's solar transmissivity, and human skin's spectral absorptivity by each wavelength, and then summing up the products. Then, the spectral integral absorptivity of human skin would be the final solar radiation absorbed by occupants over the total solar radiation incident on the human surfaces. Eq. 2 presents the calculation formula, and the propagation of solar radiation in the whole process is depicted in Fig. 2.

$$T_{sol_s} = \frac{\sum_{\lambda_1}^{\lambda_2} S \cdot T_{sol\_spe} d\lambda}{\sum_{\lambda_1}^{\lambda_2} S d\lambda}$$

$$\alpha_s = \frac{\sum_{\lambda_1}^{\lambda_2} S \cdot T_{sol\_spe} \cdot \alpha_{spe} d\lambda}{\sum_{\lambda_1}^{\lambda_2} S \cdot T_{sol\_spe} d\lambda} \quad (\text{eq. 2})$$

where, S is the spectral power distribution of solar radiation;  $T_{sol\_spe}$  is the spectral solar transmissivity of the window in use;  $\alpha_{spe}$  is skin spectral absorptivity for the target occupant;  $\lambda_1$  is the minimum wavelength in the calculation;  $\lambda_2$  is the maximum wavelength.

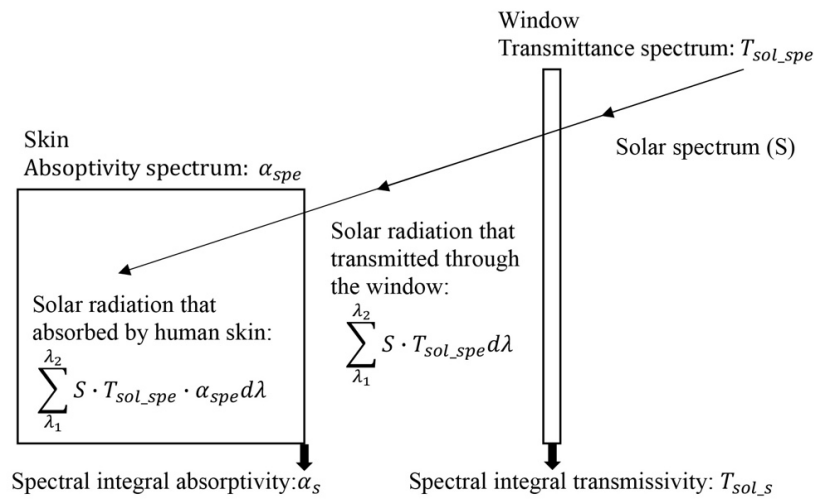


Fig. 2: The propagation of solar radiation into the human skin

## 2.2 Verification method

To verify the significance of using the proposed spectral integral method in the estimation of thermal comfort, different solar spectra were explored in this work. The variation of solar spectra at different times of day were taken into account. Thus, solar irradiance data at different times of day from 08:00 to 18:00 with an interval of one hour on June 1<sup>st</sup> in Denver, Colorado, was used. Next, we calculated the MRT delta and PMV values at different times of day by using the constant method and the spectral integral method, respectively. As a consequence, the results under the constant and spectral integral methods were compared.

Except for the spectral-related parameters, other parameters related to boundary conditions and situational contexts were maintained constant in all the calculations. The assumed values of the parameters unrelated to

spectrum variation are listed in Tabs. 1 and 2. Specifically, the parameters in Tab. 1 were adopted to calculate MRT delta, and those in Tab. 2 were set for accomplishing the calculation of PMV.

Tab. 1: The assumed values for not spectral-related parameters in calculating MRT delta

Posture	SHARP (°)	$f_{svv}$	$f_{bes}$
Seated	0	0.2	0.3

Tab. 2: The assumed values for not spectral-related parameters in calculating PMV

Air temperature (°C)	MRT (°C)	Air speed (m/s)	Relative humidity (%)	Metabolic rate (met)	Clothing level (clo)
25	25	0.1	50	1	0.6

### 3. Results

#### 3.1. Mean radiant temperature delta

The direct beam (normal) solar irradiance ( $I_{dir}$ ) and solar altitude angle ( $\beta$ ) primarily shape the solar spectra at different times of day. A single-pane window with transmittance ( $T_{sol\_c}$ ) of 0.554 was selected in this case study. The color of the human skin in the calculation was assumed white, which has a suggested absorptivity ( $\alpha_c$ ) of 0.570. The spectral transmissivity ( $T_{sol\_s}$ ) and absorptivity ( $\alpha_s$ ) were calculated by eq. 2. The constant ( $MRT\Delta_c$ ) and spectral integral ( $MRT\Delta_s$ ) MRT delta values were calculated by the above parameters and assumed parameters in Tab. 1. Hence, Tab. 3 summarizes the results of the calculation and also shows the differences between the constant and spectral integral MRT delta values ( $MRT\Delta_{diff}$ ).

Tab. 3: Constant and spectral integral mean radiant temperature values

Time	$I_{dir}$ (W/m <sup>2</sup> )	$\beta$	$T_{sol\_c}$	$T_{sol\_s}$	$\alpha_c$	$\alpha_s$	$MRT\Delta_c$	$MRT\Delta_s$	$MRT\Delta_{diff}$
08:00	468	14.4	0.554	0.5786	0.570	0.5746	3.8	4.0	0.2
09:00	669	25.6		0.5792		0.5777	5.6	5.9	0.3
10:00	622	37.1		0.5820		0.5798	5.5	5.8	0.3
11:00	914	48.5		0.5798		0.5787	8.3	8.8	0.5
12:00	922	59.4		0.5798		0.5774	8.3	8.8	0.5
13:00	115	68.5		0.5900		0.5731	1.0	1.1	0.1
14:00	448	72.4		0.5819		0.5757	4.0	4.2	0.2
15:00	745	68.0		0.5816		0.5785	6.6	7.1	0.5
16:00	399	58.7		0.5838		0.5783	3.6	3.8	0.2
17:00	435	47.7		0.5831		0.5760	4.0	4.2	0.2
18:00	116	36.3	0.6069	0.5934	1.0	1.2	0.2		

Note:  $MRT\Delta_{diff} = MRT\Delta_s - MRT\Delta_c$

We could see from Tab. 3 that the differences between the constant and spectral integral transmissivities have a magnitude of 0.01. The differences between the constant and spectral integral absorptivities have a magnitude of 0.001. The differences are minor, while the aggregated variances between the constant and spectral integral MRT deltas could be ranging from 0.1 to 0.5, which may have significant influences on thermal comfort.

The  $MRT\Delta_{diff}$  values are different across the time of day. To understand the factors that influence the

$MRT\Delta_{diff}$ , we used linear regression to find out significant predictors for  $MRT\Delta_{diff}$ . The  $I_{dir}$  and  $\sin(\beta)$  from Tab. 3, which we assumed to be the factors, were used to perform the regression. The parameters of  $I_{dir}$  ( $p$  value =  $4.58 \times 10^{-5}$ ) and  $\sin(\beta)$  ( $p$  value = 0.047) are all significant to predict the  $MRT\Delta_{diff}$ . The R-squared value 0.89 of the linear regression is also acceptable. Thus, we could conclude that the  $MRT\Delta_{diff}$  is mainly dependent on the intensity of the solar irradiance and the altitude angle of the solar.

### 3.2 Predicted Mean Vote

The PMV values were calculated by processing the MRT delta values and assumed parameters in Tab. 2. MRT delta values were added up to the initial MRT values to get new MRT values. We then used the new MRT values to derive the PMV values in both constant and spectral integral methods. The calculation results are shown in Tab. 4.

Tab. 4: Constant and spectral integral Predicted Mean Vote

PMV	8:00	9:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00
$PMV_c$	0.46	0.77	0.75	1.23	1.23	-0.01	0.49	0.94	0.43	0.49	-0.01
$PMV_s$	0.49	0.82	0.8	1.32	1.32	0	0.53	1.03	0.46	0.53	0.02
$PMV_{diff}$	0.03	0.05	0.05	0.09	0.09	0.01	0.04	0.09	0.03	0.04	0.03

Note:  $PMV_{diff} = PMV_s - PMV_c$

The differences between constant and spectral integral PMV values ranged from 0.01 to 0.09. The results are also plotted within two zones in Fig. 3. The zone in gray is the thermal comfort zone with the PMV values between -0.5 and 0.5, while in red is the region represents that occupants may feel uncomfortably warm. Furthermore, when it comes to the accurate comparison of the comfort levels, the region that the PMV values locate in becomes more important than the differences of the PMV values under the constant and spectral integral methods.

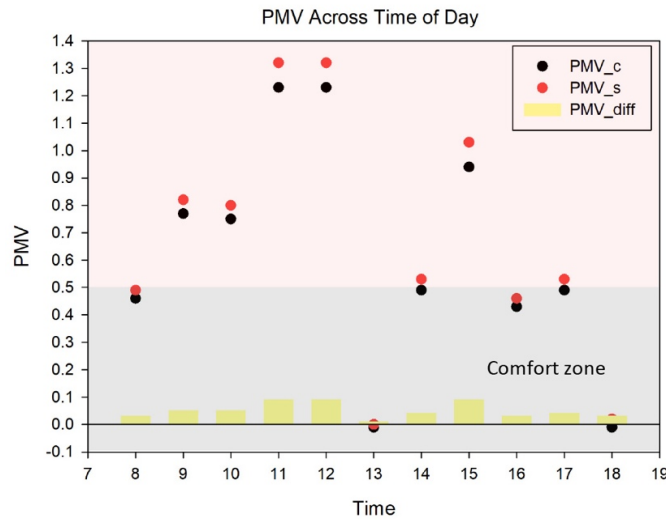


Fig. 3: The PMV values and PMV differences across times of day

At 14:00 and 17:00, the  $PMV_{diff}$  values were relatively minor, but different conclusions of the thermal comfort level are drawn for constant and spectral integral methods. The occupants are considered thermal comfort under the constant method, and uncomfortably warm under the spectral integral method. At 11:00 and 12:00, the  $PMV_{diff}$  values are relatively large, but they have the same thermal comfort level, which is both uncomfortably warm. Fig. 3 also states that the alteration of the thermal comfort level only happens at specific locations of the PMV values. As the spectral integral method is more complicated than the simplified constant method, the constant method is firstly suggested unless it is necessary to use the spectral integral method. In other words, the spectral integral method is only suggested to replace the constant method in certain cases. Thus, we should be able to determine the cases of constant PMV values that thermal comfort level might be

changed under the corresponding spectral integral method.

Another phenomenon that worth mentioning is that the  $PMV_{diff}$  values are all positive, which means the PMV values calculated with the constant method are underestimated the actual solar effects on thermal comfort. This could be problematic if a higher absorptivity is gained, such as the situations with a more intense solar irradiance, more transparent glazing, or higher skin absorptivity.

A simple linear regression modeling was also carried out to explore the factors that mainly influence the  $PMV_{diff}$ . The  $I_{dir}$  and  $\sin(\beta)$  were assumed to be the factor in this analysis, and both of these parameters are significant, with p values  $1.36 \times 10^{-5}$  and 0.018 for  $I_{dir}$  and  $\sin(\beta)$ , respectively, to predict the  $PMV_{diff}$ . The R-squared value of the linear regression is 0.92, which looks reasonable. Thus, we could conclude that the  $PMV_{diff}$  is also related to the intensity of the solar irradiance and the altitude angle of the solar.

## 5. Conclusions

The spectral integral method was obtained by using spectral related solar transmissivity and absorptivity, which was calculated by eq. 2, instead of constant values provided or suggested.

By using the spectral integral method, the transmissivities were all higher than those using the constant method with a magnitude of 0.01. For absorptivities, the higher amount was about a magnitude of 0.001. The differences of transmissivities and absorptivities caused 0.1 magnitude differences in MRT delta values, and then PMV differences between the two methods had a magnitude of 0.01. The final differences in PMV values were minor but not negligible, as the actual indoor thermal comfort determined by the PMV values could be altered by using the spectral integral method. Thus, we verified that the spectral integral method becomes a necessity when the thermal comfort analysis is involved in solar effects, especially in some specific scenarios. Furthermore, we observed that the differences of the MRT delta and PMV values yielded by the constant and spectral integral methods were significantly influenced by the solar intensity that incident on the window and the solar altitude angle. These two parameters could be practical and helpful to appreciate the necessity of the spectral integral method. More guidelines are needed to judge the necessity of the spectral integral method so that we could determine the method to use before the calculation. Future work will form a few steps that could be directly used to give a preliminary judgment.

## 6. Acknowledgments

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