AN EVALUATION METHOD FOR DESIGNING COST-EFFICIENT SINGLE AXIS PV-TRACKERS

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1. Introduction

The requirements on installation of solar trackers are not only to maximum the received solar energy but also to reduce the invested cost. With combination of increased energy, lower installation cost and lower operation complexity, single axis trackers are widely applied in silicon-based PV-panel systems. Among the mentioned advantages, maximization of the generated PV-power is always first priority for the design task to realize. For single axis tracker, however, the daily variation of the angle of incidence between incoming light and the photovoltaic panel is various due to the different values of the design parameters. On the other hand, the sunshine rate affects also the generated power. How to select a suitable

There are various works on evaluation of the performance of different PV-tracking systems based on a numerical or experimental approach. For example, Abdallah (2003) has compared different types of tracking PV-system experimentally. Chang (2009) compared also the PV-power gain of the single-axis tracker in azimuth tracking and the fixed type systematically. Koussa et al. (2011) proposed an approach to improve the performance of PV-system by comparing five different configurations of solar trackers with two fixed panels under consideration of different sky condition. Lorenzo et al. (2002) has demonstrated an approach of determining the produced energy of another type of single axis tracker, i.e. azimuth tracker, under consideration of shadowing effect. Lubitz simulated solar radiation on fixed, azimuth tracking and two axis tracking PV-panels at 217 diverse locations in USA by utilizing the Perez radiation model with hourly typical meteorological year (TMY3) data. Padovan and Col (2010) proposed new measurements of global and diffuse solar irradiance on the horizontal plane and global irradiance on tilted PV-panels with a good accuracy.

The aim of the paper is thus to propose a simple evaluation method for selecting suitable design parameters of the single axis PV-trackers under consideration of the environmental influence at the installation place to obtain the maximum generated PV-power. Especially how the design parameters affect the gain of energy is also discussed.

2. Mathematic Models

2.1 Cumulative hourly PV-power

In order to evaluate the different tracker design, a generalized equation for calculation of the cumulative hourly PV-power Q_{total} in one year is derived based on Şen (2008) as the following equation,

$$Q_{\text{total}} = \left\{ \sum_{M=1}^{12} f_{\text{SS}}(M) \cdot \sum_{N=N_{\text{a}}(M)}^{N_{\text{e}}(M)} \sum_{t=6\text{h}}^{t=18\text{h}} I_0 \cdot f_{\varepsilon}(\Omega) \cdot f_{\xi}(\Theta_i) \cdot f_{\text{d}}(m) \right\} \cdot P_{\text{e}} \ [\text{MJ/m}^2]. \qquad \text{(eq. 1)}$$

The parameters in the equation can be divided into four types:

- **constant parameters** are invariant parameter in calculating the solar power, including the energy conversion efficiency of the solar cells P_{e} and the solar constant I_0 (=1367 W/m^2 or 4.921 MJ/m^{2}h);
- **environment related parameters** are the parameters for solar irradiation. There are two influence factors, the astronomical factor $f_{\varepsilon}(\Omega)$ for the change in extraterrestrial solar radiation, and the atmospheric factor $f_{\text{d}}(m)$ for definition of atmospheric transmittance of beam radiation.
- **weather related parameter** is here the monthly sun-shine rate $f_{\text{SS}}(M)$, which is determined from the weather measured data;
- **tracker related parameter**, i.e. the hourly power decline function $f_{\xi}(\Theta_i)$ due to the angle of misalignment Θ_i. The angle Θ_i at each time point is determined according to geometric relation between the designed tracker and the position of the sun.

The above mentioned parameters are further described in the following sections corresponding to solar irradiation and misalignment of the single-axis tracker.

2.2 Solar irradiation

Solar irradiation I with unit W/m^2 is defined as incident radiant power on a unit surface,

$$I = I_0 \cdot f_\varepsilon(\Omega) \cdot f_{\mathrm{d}}(m) \ \ [\mathrm{MJ/m^2h}] \qquad \text{(eq. 2)}$$

where the astronomical factor $f_\varepsilon(N_\mathrm{d})$ is defined as

$$f_\varepsilon(\Omega) = 1 + 0.033\cos(2\pi \cdot N_\mathrm{d}/365) \qquad \text{(eq. 3)}$$

with N_d the number of the days from 1 January to a particular date in a given year;

and the atmospheric factor $f_\mathrm{d}(m)$ for definition of atmospheric transmittance of beam radiation is determined according to,

$$f_\mathrm{d}(m) = 0.56(e^{-0.65m} + e^{-0.095m}) \qquad \text{(eq. 4)}$$

with the air mass ratio m,

$$m = \sqrt{1229 + (614\sin\lambda)^2} - 614\sin\lambda \qquad \text{(eq. 5)}$$

2.3 Misalignment of the single-axis tracker

The misalignment of the PV-panel on the tracker to the solar beam is essential to determine for evaluation of the generated PV-energy. In case of the misaligned PV-panel, the generated PV-energy will be decreased according to the equation

$$I_\xi = I \cdot \cos\xi \qquad \text{(eq. 6)}$$

The instantaneous incident angle ξ of solar beam upon the PV-panel can be determined by using the inner-product of the two vectors, namely the normal vector of the PV-panel $\mathbf{N}_{\mathrm{Tracker}}$ and the vector for the solar beam $\mathbf{N}_{\mathrm{Sun}}$ (Tsai et al., 2009), **Fig. 1**,

$$\cos\zeta = \frac{{}_\mathrm{T}N_\mathrm{Sun} \cdot {}_\mathrm{T}N_\mathrm{Tracker}}{\left|{}_\mathrm{T}N_\mathrm{Sun}\right|\left|{}_\mathrm{T}N_\mathrm{Tracker}\right|} \qquad \text{(eq. 7)}$$

The astronomical relation of the sun and the earth is simplified as a spherical trigonometric model for solving the vector of the solar beam. The essential coordinate systems are considered for the derivation: (a) ecliptic coordinate system $S_E(x_E, y_E, z_E)$, (b) equatorial coordinate system $S_C(x_C, y_C, z_C)$ and (c) earth coordinate system, **Fig. 2**. The related coordinate systems are defined as the following:

- $S_E(x_E, y_E, z_E)$: is fixed on ecliptic plane, where z_E axis is normal the plane, vernal equinox is on the x_E axis, and the rotation axis of the earth is inclined to z_E axis with an angle α.
- $S_C(x_C, y_C, z_C)$: is fixed on equatorial plane. z_F axis is not only perpendicular to the equatorial plane, but also coincided with the rotating axis. x_C axis is also coincided with x_E axis.
- $S_D(x_D, y_D, z_D)$: is a moving system with the movement of the sun. z_D axis is coincided with z_C axis, and both the centers of the sun and the earth fall on the x_D-z_D-plane.
- $S_T(x_T, y_T, z_T)$: is fixed on the earth. z_T axis is coincided with z_D axis, and an angle γ exists between x_T and x_D axis due to rotation of the earth. If x_T axis and x_D axis are coincided, i.e. $\gamma = 0$, the time is defined as 12:00 (noon).
- $S_G(x_G, y_G, z_G)$: is fixed on the ground plane, where z_G axis is directed to the north and x_G axis to the west.

From the previous work of Tsai et al. (2009) the vector of the solar beam can be expressed as

$$
{}_{\mathrm{T}}\mathbf{N}_{\mathrm{Sun}} = \mathbf{R}_{\mathrm{TD}}\,{}_{\mathrm{D}}\mathbf{N}_{\mathrm{Sun}}
= \begin{bmatrix} \cos\gamma\sqrt{\cos^2\Omega+\cos^2\alpha\cdot\sin^2\Omega} \\ -\sin\gamma\sqrt{\cos^2\Omega+\cos^2\alpha\cdot\sin^2\Omega} \\ \sin\alpha\cdot\sin\Omega \end{bmatrix} \qquad \text{(eq. 8)}
$$

with

$$
\Omega = \frac{360^\circ}{365}\left(N_d - 80\right). \qquad \text{(eq. 9)}
$$

The normal vector to the PV-panel is derived from the coordinate transformation as

$$
{}_{\mathrm{T}}\mathbf{N}_{\mathrm{Tracker}} = \mathbf{R}_{\mathrm{TG}}\mathbf{R}_{\mathrm{GF}}\mathbf{R}_{\mathrm{FR}}\,{}_{\mathrm{R}}\mathbf{N}_{\mathrm{Tracker}}
= \begin{bmatrix} \cos(\delta-\theta) & 0 & -\sin(\delta-\theta) \\ 0 & 1 & 0 \\ \sin(\delta-\theta) & 0 & \cos(\delta-\theta) \end{bmatrix} \cdot
\begin{bmatrix} \cos\psi\cos\varphi & \sin\psi & -\cos\psi\sin\varphi \\ -\sin\psi\cos\varphi & \cos\psi & \sin\psi\sin\varphi \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix}
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(eq. 10)}
$$

or

$$
{}_{\mathrm{T}}\mathbf{N}_{\mathrm{Tracker}} = \begin{bmatrix} \cos(\delta-\theta)\cos\psi\cos\varphi + \sin(\delta-\theta)\sin\varphi \\ -\sin\psi\cos\varphi \\ \sin(\delta-\theta)\cos\psi\cos\varphi - \cos(\delta-\theta)\sin\varphi \end{bmatrix} \qquad \text{(eq. 11)}
$$

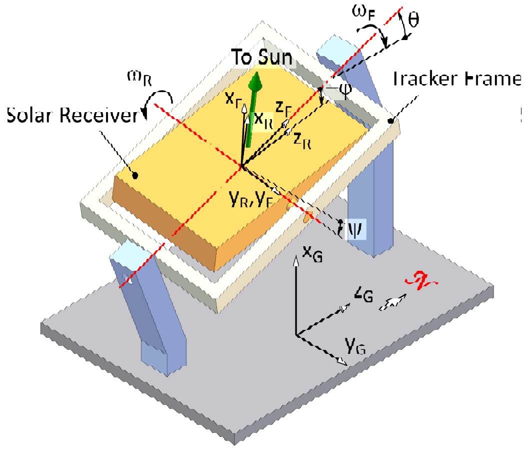


Fig. 1 Definition of the dual-axis solar tracker in roll-tilt type

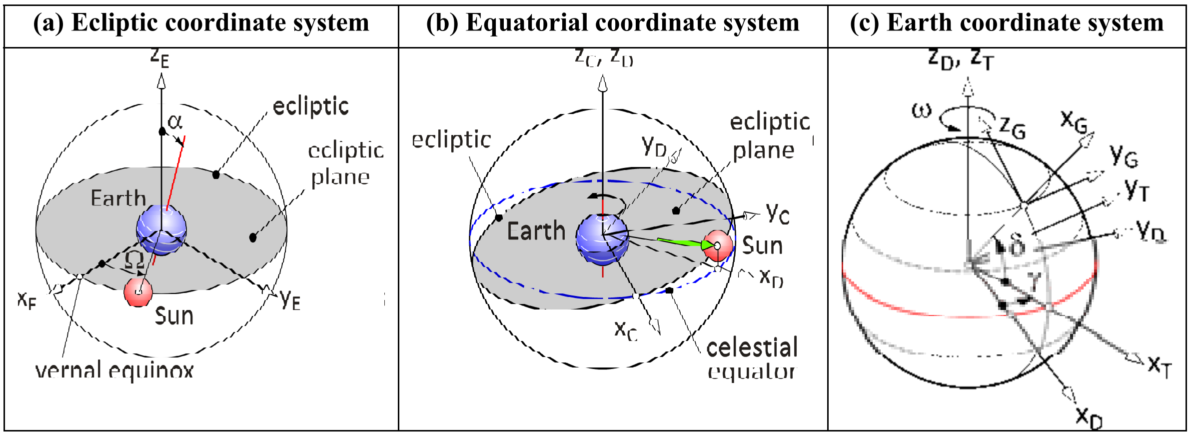
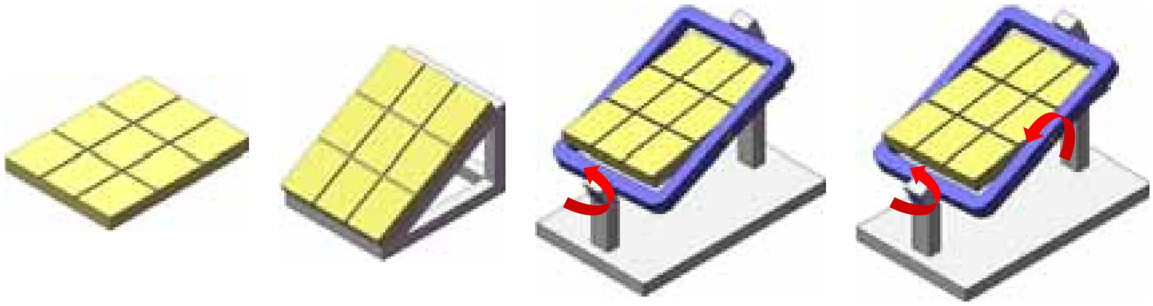

(a) Ecliptic coordinate system
(b) Equatorial coordinate system
(c) Earth coordinate system

Fig. 2 Coordinate system for derivation of the solar beam

3. Analysis Results

In the following, different types of tracker, namely horizontally fixed panel, inclined fixed panel, single axis (east-west) and dual-axis (tip-tilt type), **Fig. 3**, are analyzed. Two locations of installation are considered, Hengchun and Jhongli, which geographical locations can be found in **Fig. 4**. Some analysis works are conducted,

- Verification of the calculation approach for solar irradiation by comparison with the measured data,
- Exploration of the influence of the tracker design parameters on misalignment of the panel,
- Exploration of the influences of the tracker design parameters on the generated PV-energy,
- Optimization of the tracker design parameters for the highest generated PV-energy.



Fixed (Horizontal) Fixed (Inclined) Single-Axis Dual-Axis

Fig. 3 Different types of tracker for evaluation of PV-energy

Fig. 4 Geographical locations for simulation: Jhong-li and Heng-chun

3.1 Comparison of the solar determination model with the measured value

In order to verify the model for calculation of the solar irradiation, the calculation results are compared with the measured data from the sunshine senor, installed at the National Central University, Jhongli (**Fig. 5**). Two data measured on 26.09.2009 and 28.12.2010 are compared and illustrated in **Fig. 6**. The two measured days are sky clear. It is clear to recognize that the calculation approach is reliable for determination of the solar irradiation.

Fig. 5 Sunshine Sensor (Type BF3)

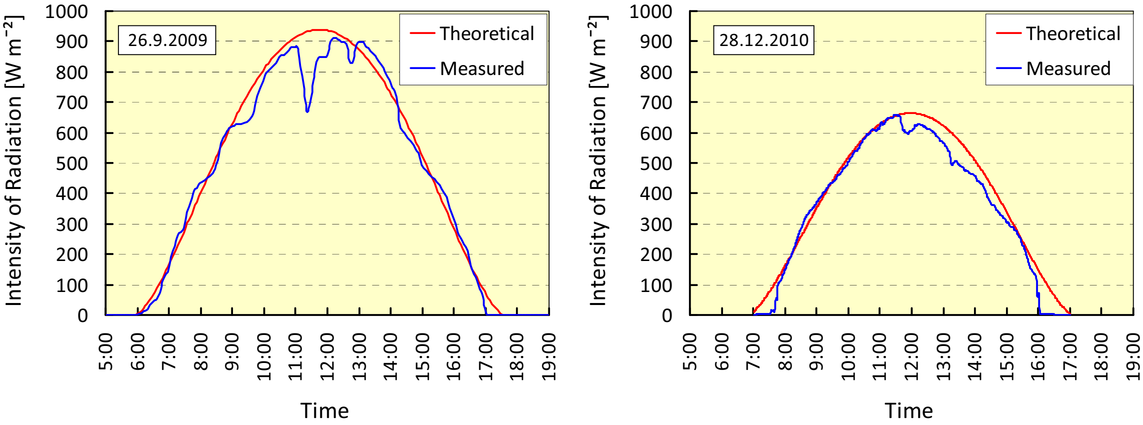


Fig. 6 Comparison of the calculated radiation with the measured data

3.2 Influence of the tracker design parameters on misalignment of the panel

The variation of the misalignment angle plays an important role for designing solar tracker. Different design parameters will cause different variation. Two parameters of the solar trackers, i.e. the tilt angle θ for inclination of the main axis and the tip angle φ for inclination of the PV-panel related to the main axis, are considered here to explore their influence on the misalignment.

Fig. 7 illustrates the variation of the misalignment angle of the PV-panel in a year due to different tip angles φ. The tilt angle θ is defined as the same with the latitude of the installation, here Jhongli. It can be recognized that the misalignment angle has an even variation if the tip angle φ is equal to zero. In this case, the misalignment angle is also equal to zero both on vernal and autumnal equinox. The misalignment angle tends to be zero on summer solstice with the tip angle φ to –23.4º, on the other hand, it is also zero on winter solstice with tip angle φ to 23.4º. Another fact is not shown here that the misalignment angle in a day is not varied if the tilt angle θ is equal to the latitude of the installation. It is thus clear that the adjustment of the tip angle can vary the misalignment angle in a year.

The effect of another parameter, the tilt angle θ, on the misalignment is illustrated in **Fig. 8**, where the tip angle φ is settled as zero. We can recognize from **Fig. 7** and **Fig. 8** that the misalignment angle is increased from morning to noon and decreased again to dusk in a day on the days near summer solstice, and in contrast, it is also decreased at first to noon and increased again in a day on the days near winter solstice, in case of the tilt angle θ smaller than the latitude of installation, here 15º. In case of a larger tilt angle θ, here 30º, we can find a contrary variation.

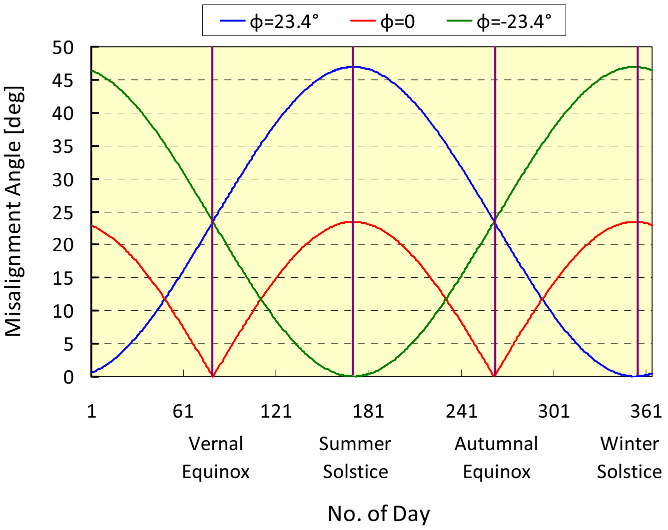


Fig. 7 **Variation of the misalignment of the PV-panel in a year due to different tip angles φ**

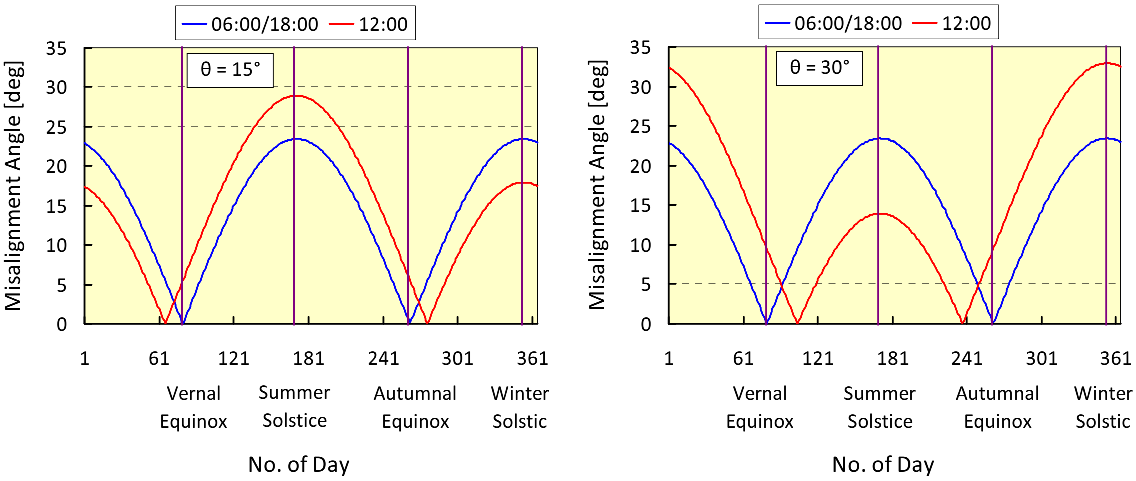


Fig. 8 **Variation of the misalignment of the PV-panel in a year and a day due to different tilt angles *θ***

3.3 Influence of the tracker design parameters on the generated PV-energy

The influence of the tilt angle θ of the main axis of the tracker and the tip angle φ of the PV-panel on the generated PV-energy is discussed based on the proposed approach. The monthly sunshine rates used for calculation are average values adopted from the database of Taiwan Central Weather Bureau based on 10-year measurement, **Fig. 9**.

The generated PV-power of the trackers in single-axis design and with various tilt angles is illustrated in **Fig. 10**, each for location in Hengchun and Jhongli, respectively. The tip angle of the PV-panel is in this case equal to 0. It is very clearly to find that a greater PV-power can be obtained for single-axis tracker or inclined-fixed PV-panel if the tilt angle is settled around the latitude of installation. However, the difference between the maximum value and the minimum value is not significant, for example, in Hengchun the factor is about 1.04 for single axis tracker, and 1.05 for inclined fixed PV-panel; in Jhongli only 1.05 for single-axis tracker, and 1.07 for inclined fixed PV-panel.

The tip angle, in contrast to the tilt angle, has a significant effect on the generated power. Because the tip angle is only considered while in designing the single-axis tracker, there are three type of tracker/panel for comparison in **Fig. 11**. It is obvious that the tip angle has also similar effect as the tilt angle. A larger PV-energy can be generated if the tip angle is designed near zero. However, the ratio of maximum to minimum value is greater than the case of the tilt angle, for example, 1.08 in Hengchun and 1.09 in Jhongli.

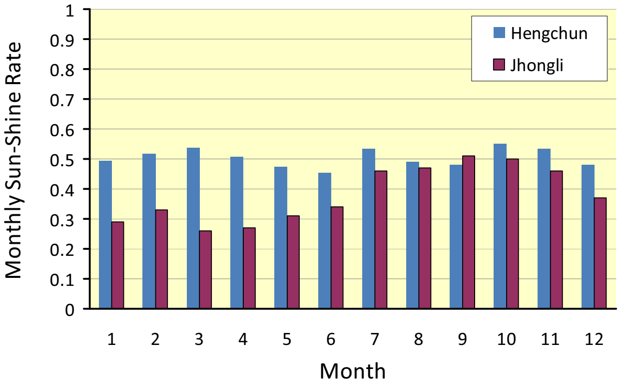


Fig. 9 Average monthly sun-shine rate

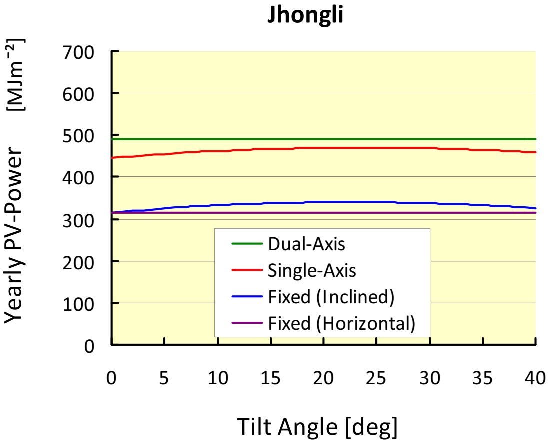


Fig. 10 Influence of the tilt-angle θ on the generated PV-power

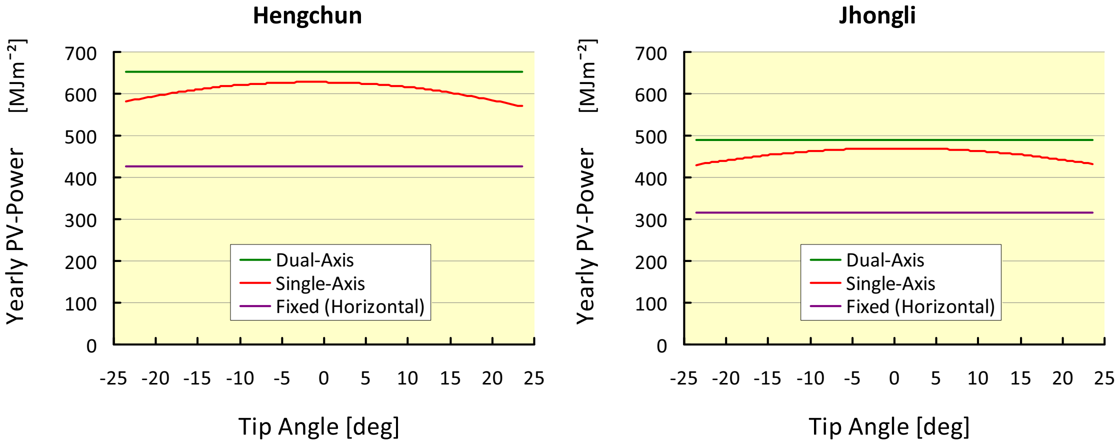


Fig. 11 Influence of the tip-angle φ on the generated PV-power

3.4 Optimization of the tracker design parameters for the highest generated PV-energy

In order to evaluate the maximum possibly generated PV-power, an optimization method, here Powell method, is also introduced in the paper (Press et al. 2007). The optimal parameters based on the proposed method for the single axis tracker and inclined fixed panel are calculated as listed in **Table 1**, each for the locations in Hengchun and Jhongli, respectively. The single axis tracker with the tilt angle 30.5° and the dip angle –7.4° can generate maximum PV-power in Hengchun, and with the tilt angle 33.4° and the dip angle –8.3° in Jhongli. On the other hand, the PV-panel of the fixed system should be inclined 20.3° in Hengchun and 21.9° in Jhongli to the south for generation of the maximum power. The yearly generated PV-power of tracker or fixed PV-system can be compared clearly with each other from **Fig. 12.** In comparison with the inclined fixed PV-system, the PV-power gain from the dual-axis tracker increases up to 43.7% both in Hengchun and Jhongli, and for the single-axis tracker 38.1% in Hengchun and 38.2% in Jhongli. The results reveal also the facts that the maximum power of the single axis tracker can be obtained, independent on the location, but under fulfillment of the following strategies:

- the misalignment angle on morning and evening of a given day in summer is smaller, and in winter greater;
- the average misalignment angle of a given day around the vernal and autumnal equinox is near to zero.

The monthly difference of the PV-energy between the mentioned PV-systems is illustrated in Fig. 13. It is obvious to find that the PV-energy obtained by the single-axis tracker is smaller than that by the dual-axis tracker in winter and in summer. On the other hand the inclined fixed PV-system generates less energy than the horizontal fixed PV-system from April to September, and more energy in the other months.

Table 1 Optimal Design Parameter for various types of trackers and their maximum generated yearly PV-power

	Hengchun		**Jhongli**	
Type of Tracker	Optimal Parameter	Yearly PV-Power	Optimal Parameter	Yearly PV-Power
Horizontal Fixed	--	427 MJ/m^2	--	316 MJ/m^2
Inclined-Fixed	$\theta = 20.3°$	455 MJ/m^2	$\theta = 21.9°$	341 MJ/m^2
Single Axis	$\theta = 30.5°$; $\varphi = -7.4°$	628 MJ/m^2	$\theta = 33.4°$; $\varphi = -8.3°$	471 MJ/m^2
Dual Axis	--	653 MJ/m^2	--	490 MJ/m^2

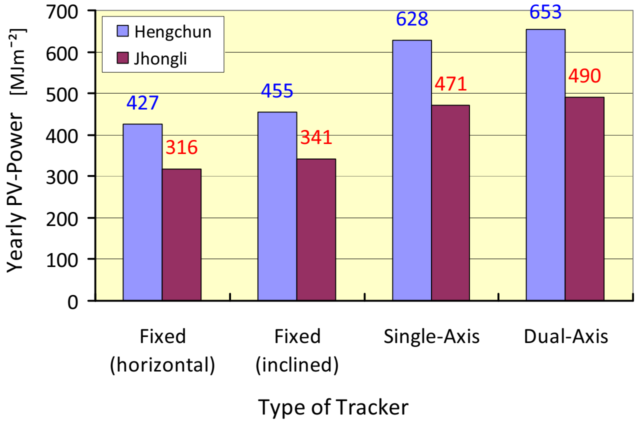


Fig. 12 Yearly generated PV-power of different types of solar tracker with the optimal parameters

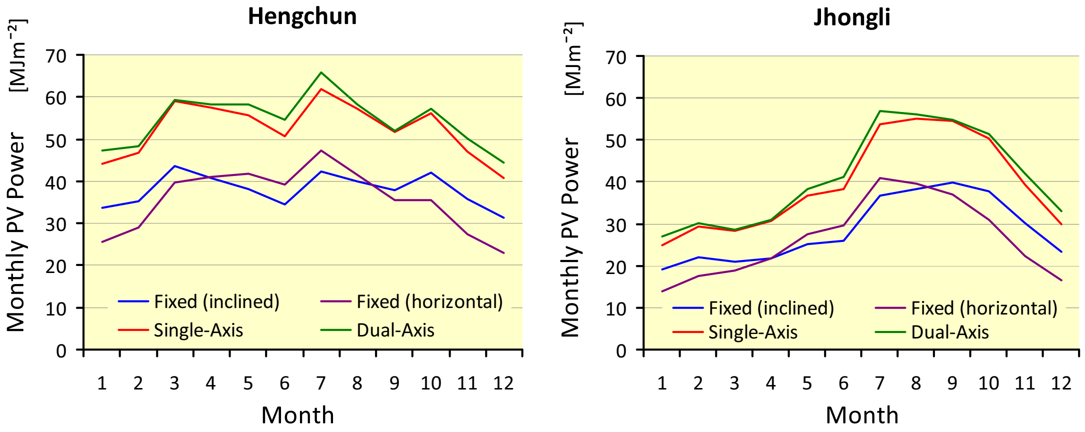


Fig. 13 Monthly PV-power of various solar trackers or fixed PV systems

4. Conclusions

The proposed method is applied successfully to analyze four types of PV-system in Hengchun and Jhongli, Taiwan. The results enable us to draw the following conclusions:

- The calculated PV-energy based on the proposed approach has a good agreement with the measured data from a sunshine sensor.
- The tilt angle of the main axis of the single-axis tracker affects mainly the distribution of the misalignment angle in a given day, while the tip angle changes the daily variation of the misalignment angle.
- The analysis results show cost benefit of the single axis tracker that its generated PV-power with optimal design is only 3.9% less than the dual axis tracker and 36.3% more than the fixed PV-system.

5. Acknowledgement

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6. References

Abdallah, S., 2003. The effect of using sun tracking systems on the voltage–current characteristics and power generation of flat plate photovoltaics. Energy Conversion and Management 45, 1671–1679.

Chang, T. P., 2009. The gain of single-axis tracked panel according to extraterrestrial radiation. Applied Energy 86 , 1074–1079.

Koussa, M., Cheknane, A., Hadji, S., Haddadi, M., Noureddine, S., 2011. Measured and modelled improvement in solar energy yield from flat plate photovoltaic systems utilizing different tracking systems and under a range of environmental conditions. Applied Energy 88, 1756–1771.

Lorenzo1, E., Perez, M., Ezpeleta, A., and Acedo, J., 2002. Design of Tracking Photovoltaic Systems with a Single Vertical Axis. Progress in Photovoltaics: Research and Application 10, 533–543

Lubitz, W. D., 2011 Effect of manual tilt adjustments on incident irradiance on fixed and tracking solar panels. Applied Energy 88, 1710–1719

Padovan, A., Col, D. D., 2010. Measurement and modeling of solar irradiance components on horizontal and tilted planes. Solar Energy 84, 2068-2084.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P., 2007. Numerical Recipes, 3rd Edition, Cambridge University Press.

Şen, Zekai, 2008. Solar Energy Fundamentals and Modeling Techniques: Atmosphere, Environment, Climate Change and Renewable Energy, Springer-Verlag, London.

Tsai, S.-J., Chen, P.-L., Guo, R.-J., Yang, T.-P., Wu, J.-C., 2009. Analysis of influences of assembly errors on the tracking precision of a low profile solar tracker. Proceedings of 26th CSME National Conference, National Cheng-Kung University, Tainan, Taiwan.