OPTIMISING THE INCLINATION OF SOLAR PANELS TAKING ENERGY DEMANDS INTO CONSIDERATION

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1. Abstract

Ideally, the angle of inclination of non-tracking solar panels, whether of the photovoltaic variety or solar thermal, should be determined in relation to the energy demand curve they are meant to satisfy. This is not always the case since it is common practice to install solar panels at a particular angle, depending on the geographical location of the site, irrespective of the demand curve. For example in Malta, installers have for many years been fixing panels for solar water heating at an angle of inclination of 45 degrees to the horizontal, since Malta is situated at a latitude of 36 degrees North.

The authors of this paper have looked at a number of different demand curve scenarios, namely for maximum output, constant energy demand and constant hot water demand for the whole year and for winter and summer months. A computer program was written to simulate the solar radiation received on planes set at different angles of inclination and azimuth. A factor was introduced to correct for the effect of clouds, water vapour and particles in the atmosphere. The factor for Malta was derived using data downloaded from the website of the Joint Research Centre (JRC) of the European Commission. The program was then able to calculate the optimal angle of inclination for the different scenarios considered. To calculate the optimal angle for demand matching, two approaches were considered, one based on the best fit between demand and insolation using the lowest standard deviation and the other based on minimum area of collector panels.

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2. Introduction

Solar panels, whether photovoltaic or solar thermal, are either mounted on a fixed frame or made to track the movement of the sun. The latter are more efficient but incur higher capital and maintenance costs. The former are therefore more economical and simpler to operate and by far more resistant to extreme weather conditions. The fixed panels are therefore very popular. Their lower efficiency can be compensated by installing panels having a larger surface area.

However, since their inclination and orientation is fixed, the system designer or installer must at the outset decide these two parameters, which once set will determine the energy output of the panels. The optimum angle of inclination and the orientation (azimuth) of the panels will depend on the energy demand curve that the panels are designed to satisfy. For example, a system that is expected to supply more energy in winter will be set at a higher angle of inclination than one that is required to supply more energy in summer.

This paper discusses this issue by looking at a number of possible scenarios, i.e. different energy demand curves, and using Malta as a case study, determines the optimum angle for each case, using an algorithm written in MATLAB.

This is a different approach from that adopted by Boland and Zekanovic (2008) who developed a methodology for determining the optimal orientation for photovoltaic cells to match a particular electricity demand, using Adelaide as a case study, and using stochastic programming. It is interesting to note that Malta and Adelaide have almost the same latitude, 35°50′ and 34°55′ respectively, albeit Malta is North and Adelaide South of the equator. They concluded that the optimal orientation of PV panels in Adelaide is 33 degrees from the horizontal and rotated 6 degrees West of North when using deterministic inputs. Considering summer only, the panels should be tilted at 16 degrees and oriented 24 degrees West of North.

Using stochastic inputs, the optimal orientation for load matching is either 14.8 or 15.4 degrees to the horizontal and 18 degrees West of North.

Yang and Lu (2007) developed a mathematical model for calculating the optimum tilt and azimuth angles for building-integrated photovoltaic applications in Hong Kong (latitude 22°15′N) on a yearly, seasonal and monthly basis. They based their model on a correlation between the hourly diffuse ratio and clearness index for Hong Kong developed by Yik et al. (1995). They found that the optimum tilt angle for a south facing azimuth is 20 degrees. In winter, this should be 41 degrees.

Kumar et al. (2011) calculated the solar radiation incident on a PV array in Khaktar Kalan in the Punjab (latitude of 31.6°) using a computer model, and using a relation for the clearness index as a function of latitude, altitude and maximum and minimum temperature of the site. The results of their calculations showed that the optimal angle over a year was 30.61°, for the winter months (December, January and February) 57.48°, for spring (March, April and May) 18.16°, for summer (June, July and August) 2.83°, for autumn (September, October, November) 43.67°. They also calculated the collected solar energy of the array if the tilt angle is adjusted periodically.

3. Computer Model

The computer model for the work described in this paper is written in MATLAB and can be said to be built up of two parts. The first calculates the solar irradiation based on the equations listed below and the second determines the optimal angle of inclination of the panels for different cases. The computer program can also account for different azimuth angles, but in this paper, only south facing units are considered. The equations for determining the irradiation are taken from Duffie and Beckman (2006), Kreith and Kreider (1978) and Zekai (2008).

The total radiation (G_t) incident on a plane inclined at an angle β to the horizontal is given by:

$$G_t = G_b + G_d + G_r \quad \text{(eq. 1)}$$

G_b is the beam radiation

G_d is the diffuse radiation

G_r is the reflected radiation

Let θ be the angle of incidence of the beam radiation on the surface,

δ the declination of the sun, i.e. its angular position at solar noon,

φ the latitude of the site in question,

γ the surface azimuth angle and

ω the hour angle, i.e. the angular displacement of the sun east or west of the local meridian

Then

$$\cos\theta = \sin\delta \sin\varphi \cos\beta - \sin\delta \cos\varphi \sin\beta \cos\gamma + \cos\delta \cos\varphi \cos\beta \cos\omega + \cos\delta \sin\varphi \sin\beta \cos\gamma \cos\omega + \cos\delta \sin\beta \sin\gamma \sin\omega \quad \text{(eq. 2)}$$

$$\delta = 0.006918 - 0.39912\cos B + 0.07257\sin B - 0.006758\cos 2b + 0.000907\sin B - 0.002697\cos 3B + 0.00148\sin 3B \quad \text{(eq. 3)}$$

where $B = (N-1)360/365.242$ (eq. 4)

N is the n^{th} day in the year with January 1^{st} being given by N = 1.

$$\omega = 15(\text{solar time} - 12) \quad \text{(eq. 5)}$$

The relationship between solar time and standard time is given by:

$$\text{Solar time} = \text{standard time} + 4(L_{st} - L_{loc}) + E \qquad \text{(eq. 6)}$$

where L_{st} is the standard meridian for the local time zone

L_{loc} is the longitude of the location in question

and $E = 229.18(0.000075 + 0.001868\cos B - 0.032077\sin B - 0.014615\cos 2B - 0.04089\sin 2B)$ (eq. 7)

The beam radiation (G_b) is given by: $G_b = G_o\tau_b\cos\theta$ (eq. 8)

G_o is the extra-terrestrial radiation i.e. the energy from the sun per unit time, incident on unit area normal to the direction of the propagation of the radiation at a mean earth-sun distance, outside of the atmosphere, and is dependent on the time of the year according to the relation:

$$G_o = G_{sc}\,(1 + 0.33\cos(360N/365)) \qquad \text{(eq. 9)}$$

where G_{sc} is the solar constant and is equal to 1367 W/m²

τ_b is the transmittance and is given by either of the following two equations:

1. Transmittance (τ_b) in clear sky (no clouds) without taking into account the effect of water vapour and particulates in the air is given by:

$$\tau_b = 0.56(e^{-0.56m} + e^{-0.95m}) \qquad \text{(eq. 10)}$$

where $m = (1229 + (614\sin\alpha)^2)^{0.5} - 614\sin\alpha$ (eq. 11)

and $\alpha = 90 - \theta_z$ (eq. 12)

and θ_z is the zenith angle and is given by:

$$\cos\theta_z = \cos\varphi\,\cos\delta\,\cos\omega + \sin\varphi\,\sin\delta \qquad \text{(eq. 13)}$$

2. Transmittance (τ_b) in clear sky (no clouds), taking into consideration the effect of water vapour and particulates in the air is given by:

$$\tau_b = a.r_a + b.r_b(-c.r_c.m) \qquad \text{(eq. 14)}$$

where $a = 0.4237 - 0.008216\,(6 - \alpha)^2$ (eq.15)

$b = 0.5055 + 0.00595\,(6.5 - \alpha)^2$ (eq. 16)

$c = 0.2711 + 0.1858\,(2.5 - \alpha)^2$ (eq. 17)

and $r_a = 0.95$ $r_b = 0.98$ $r_c = 1.02$

The diffuse radiation (G_d) is given by: $G_d = G_o\tau_d\cos\theta$ (eq.18)

where $\tau_d = 0.271 - 0.294\,\tau_b$ (eq. 19)

For the reflected radiation (G_r), the albedo factor was taken as 0.25, giving:

$$G_r = 0.25(G_{bh} + G_{dh})(1 - \cos\beta)/2 \qquad \text{(eq. 20)}$$

where G_{bh} and G_{dh} are the beam and diffuse radiation falling on a horizontal surface.

Using the above equations, it is possible to calculate the irradiation on a given surface for any angle of inclination and orientation in any given location defined by its longitude and latitude, without taking into account, however the effect of clouds. To include the effect of clouds, it is necessary to include actual data for the particular location. In this work, it was decided to use, as actual true values, the data freely available from the Photovoltaic Geographical Information System (PVGIS) of the Joint Research Centre (JRC) of the European Commission, available on their website http://re.jrc.ec.europa.eu/pvgis/. Of the two databases available, the more recent one was chosen, i.e. the one designated as Climate-SAF PVGIS.

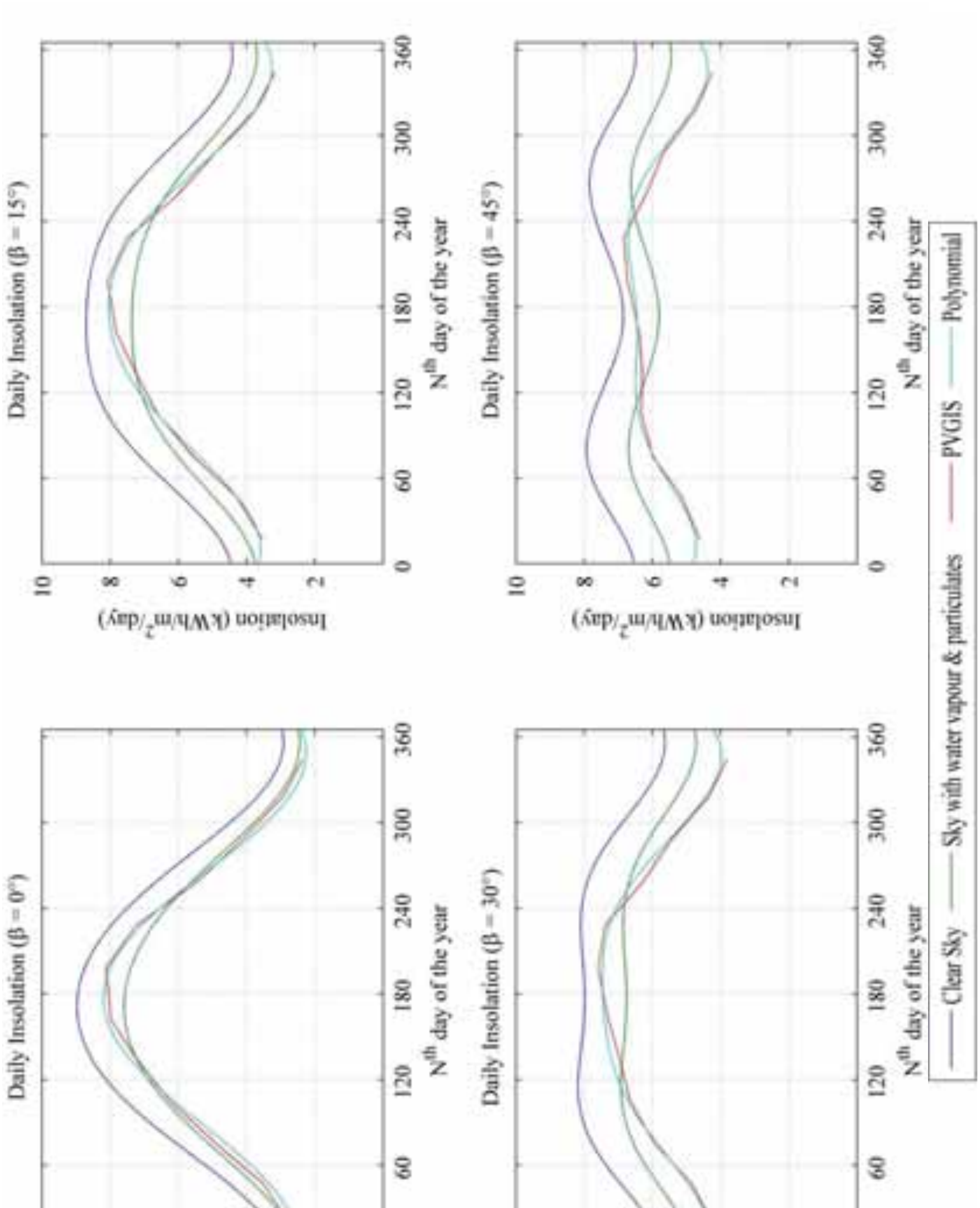

Fig 1: Daily Insolation for various angles of inclination β

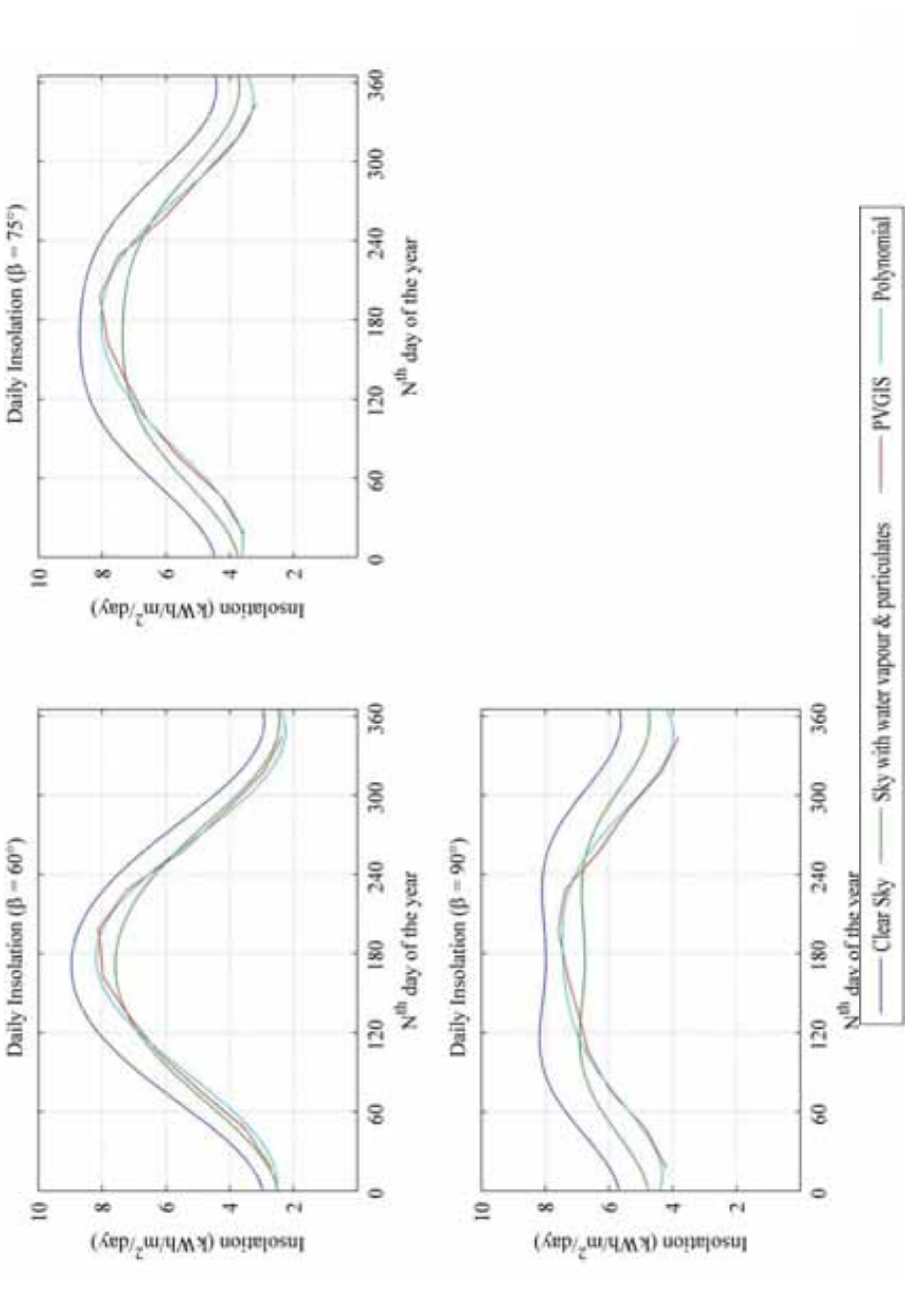

Fig. 2: Daily Insolation for various angles of inclination β

The graphs, figures 1 and 2, show the results of calculating (1) the average daily insolation for a clear sky with no water vapour, no particulates and no clouds, (2) the average daily insolation for a clear sky with no clouds but with water vapour and particulate, and (3) average daily insolation from the Climate-SAF PVGIS database, all for the location at 35.90321 degrees North and 14.48522 degrees East, which are the coordinates for the University of Malta. In all cases, the orientation is due South. The average daily insolation (H) was calculated by calculating G_t for every one minute of the day and then summing over the whole day. The graphs are plotted for every 15 degrees of inclination (β) of the panels from 0° (horizontal) to 90° (vertical).

The values from the "PVGIS" are considered as actual true values. Comparing these values with those for the "Clear Sky" condition (i.e. no clouds, no water vapour, no particulates), it can be seen that the program overestimates the insolation. This is to be expected since clouds, water vapour and particulates reduce the amount of radiation reaching the earth. Comparing the values from "PVGIS" with those for the "Sky with water vapour and particulates" (but no clouds) shows that the program underestimates the insolation for the summer months, but overestimates them in winter, since it does not allow for the effect of the clouds in winter.

A factor is therefore needed that allows for water vapour, particulates and clouds and that when applied to the values of insolation calculated for the case of clear sky with no water vapour, particulates and clouds, produces results that would be very close to the data from PVGIS. This factor can be obtained by dividing the data from "PVGIS" by that for the "Clear Sky" condition. When this was done, it was found that this factor varies with time of the year N and inclination β. The appropriate function in MATLAB was therefore used to obtain a polynomial for this factor, (denoted as F_{wpc}) as a function of N and β. The value of N was taken to be the average day of the month as per table 1 below, since the PVGIS data is only available per month. It was found that a polynomial to N^5 and β^5 gave an adjusted R-square value of 0.9805, whereas a polynomial to N^4 and β gave an adjusted R-square value of 0.9672. The latter was considered adequate for the purposes of this work and since it reduces considerably the complexity of the polynomial, it was the one adopted. The polynomial, shown in equation 21 below, was used in the program to adjust the results of the calculation by taking into account, in one factor (F_{wpc}) the effects of clouds, water vapour and particulates.

$$F_{wpc} = 0.858 - 5.631*10^{-3}N - 2.703*10^{-3}\beta + 8.155*10^{-5}N^2 + 4.662*10^{-5}N\beta - 3.546*10^{-7}N^3 - 1.861*10^{-7}N^2\beta + 4.726*10^{-10}N^4 + 1.664*10^{-10}N^3\beta \qquad \text{(eq. 21)}$$

Tab. 1: Average Day of the month (Duffie and Beckman, 2006)

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
N	17	47	75	105	135	162	198	228	258	288	318	344

Figure 3 below is a plot of equation 21, i.e. showing how the factor F_{wpc} varies with day of the year N and angle of inclination β.

Hence the insolation (H_{tot}) falling on the panel is given by:

$$H_{tot} = F_{wpc} \times H_t \qquad \text{(eq. 22)}$$

where H_t is based on the clear sky conditions using equation 10 for the transmittance through the sky without water vapour, particulates and clouds, azimuth = 0° (South) and albedo = 25%. The results for H_{tot} are shown below in figure 4 and are also included in the graphs of figures 1 and 2 (as Polynomial), and show a good agreement with the values from PVGIS. The data from PVGIS is only available for each month of the year, which explains why the plot of this data in the graphs is not a smooth curve. The value for each month was plotted for day N as per table 1 above.

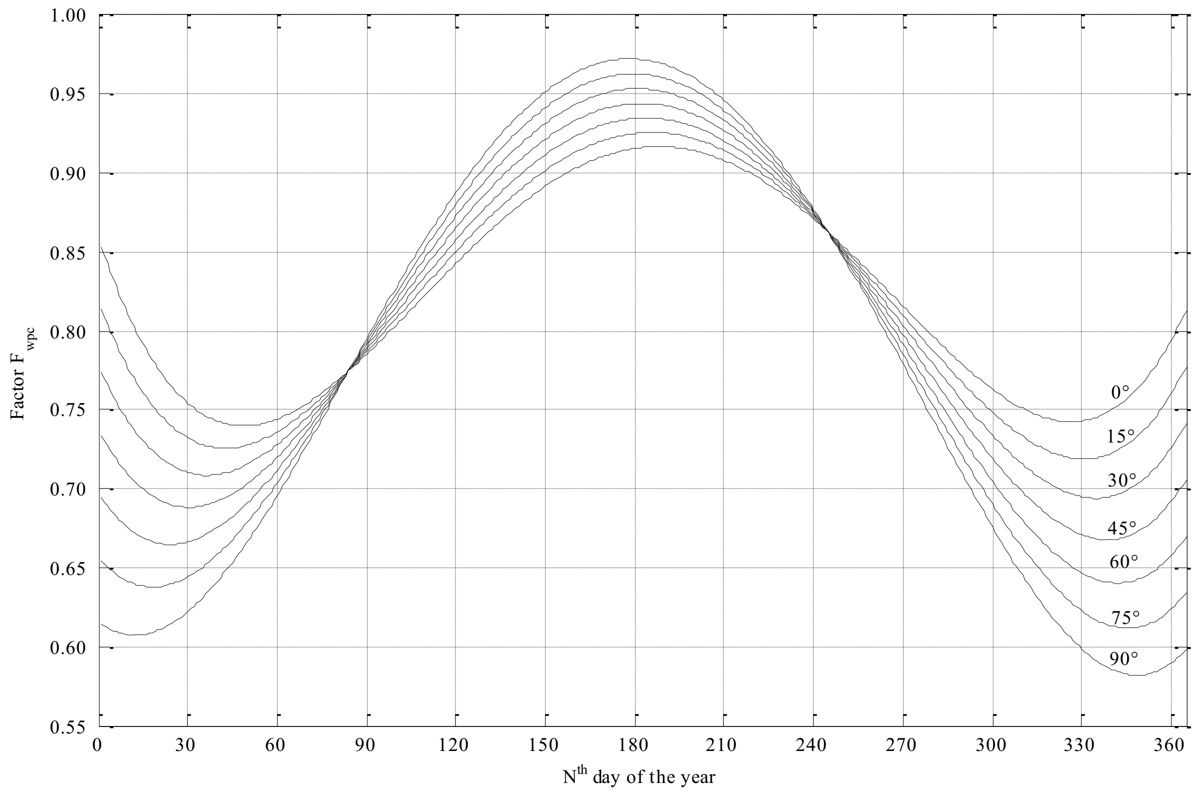

Fig. 3: The factor F_{wpc} plotted against day N of the year for different angles of inclination of the panels, β (equation 21).

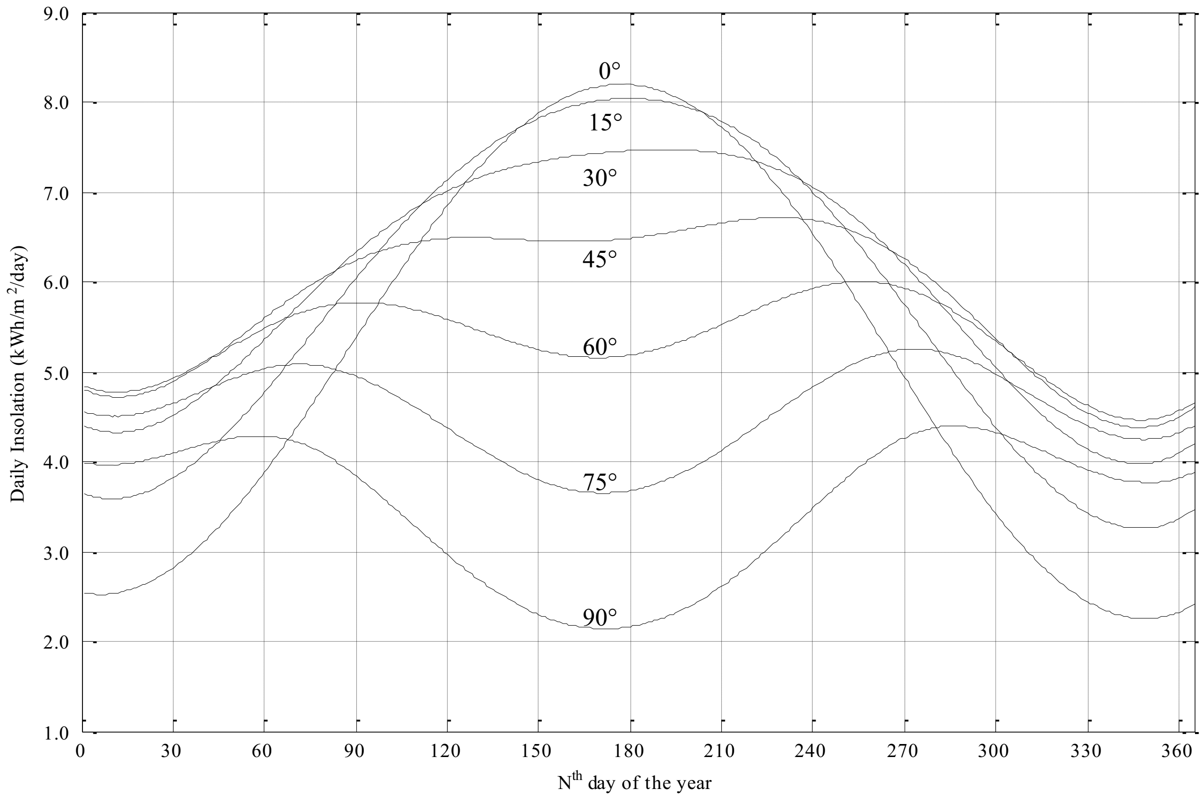

Fig. 4: The daily insolation H_{tot} plotted against day N of the year for different angles of inclination, β

The second part of the computer program calculates the optimal angle for a given demand load D, which can be characterised in terms of N, i.e. for every N_i a demand D_i can be specified for a period of time going from a specified day N_x to another specified day in the year N_y. The computer model calculates the angle at which the plane would receive the maximum amount of radiation as well as the amount of radiation for a given period, e.g. a full year or four winter months. It can also calculate the optimal angle for any demand (specified in terms of N) in two ways.

One is by calculating the standard deviation for the ratio between the radiation falling on the panel and the demand load on a daily basis for the specified period. A standard deviation is thus calculated for every one degree of inclination and the optimal angle is defined as the angle that gives the lowest standard deviation, this being the condition where the insolation curve and demand curve are closest, i. of "best fit" between the two curves.

A second method involves calculating the area of the panel required to satisfy the daily load at every angle of inclination. The maximum area is then chosen for each angle of inclination in order that the demand is satisfied on every day in the period under investigation. The optimal angle of inclination is then the one that requires the smallest of these maximum areas.

In the next section, the results of applying the program to a number of selected cases are presented. Although the program can also calculate the effect of the azimuth, for the purpose of this paper, this was in all cases taken as south facing.

4. Different Scenarios

Although many scenarios are possible, the following were the cases that were considered. All were applied to Malta (latitude 35°54'and longitude 14°29'), with the panels facing south. For another site, a new F_{wpc} factor would have to be established since this factor is site specific.

The first scenario is the most straightforward and calculated the optimal angle for maximum insolation over a period. In the second case, it was assumed that the amount of heat required on a daily basis remains constant over the period in question. The third calculated the optimal angle for the case of a solar water heater which is supplying the same amount of hot water on each day of the period in question. The amount of energy required on a daily basis varies and this was calculated by considering that the solar panels have to heat the water from the ambient temperature to 65^0 C, which is the recommended temperature to avoid Legionella. The quantity of hot water required was assumed to remain constant during the period

Three periods were considered:

1. A full year
2. Winter taken as 1st December to 31st March
3. Summer taken as 1st June to 30th September

5. Results

For the full year, the optimal angle for maximum insolation is 32° with a total insolation received by unit area of panel of 2182 kWh/m^2, i.e. a daily average of 5978Wh/m^2/day. This angle compares well with current practice in Malta of installing PV panels at 30°, and with the results from other locations at roughly the same latitude as mentioned in the Introduction. Table 2 below gathers together these results.

Tab. 2: The optimal angle for maximum insolation over a year.

Location	Latitude	Optimal Angle	Reference
Hong Kong	22° 15' N	20°	Yang and Lu
Adelaide	34° 55' S	33°	Boland and Zekanovic
Khaktar Kalan	31° 36' N	30.61°	Kumar et al.
Malta	35° 54' N	32°	This work

The optimal angle for maximum insolation received over the winter and summer months and the total insolation received at this angle were also calculated. The results are shown in table 3 below.

Tab. 3: The optimal angle of inclination for maximum insolation for different periods.

Period	Optimal angle for maximum insolation	Total insolation for the period, kWh/m^2	Daily average insolation for the period, $Wh/m^2/day$
Full year	32°	2182	5978
Winter (1st December to 31st March)	51°	619	5116
Summer (1st June to 30th September)	15°	901	7385

As expected the optimal angle for maximum insolation in winter is steeper than that for the whole year, whilst that for summer is much closer to the horizontal. The results for the other cases are shown in table 4 below.

Tab. 4: The results of the calculations for different cases.

Case	Method	Optimal angle in degrees	Total insolation for the period kWh/m^2	Daily average insolation $Wh/m^2/day$
Constant energy demand all year	Best fit	64	1880	5151
	Minimum area	56	2008	5501
Constant energy demand winter months	Best fit	90	488	4033
	Minimum area	56	616	5091
Constant energy demand summer months	Best fit	46	792	6492
	Minimum area	41	824	6754
Constant hot water all year	Best fit	73	1700	4658
	Minimum area	56	2008	5501
Constant hot water winter months	Best fit	90	488	4033
	Minimum area	56	616	5091
Constant hot water summer months	Best fit	45	799	6549
	Minimum area	41	824	6754

First of all, one notes that the two methods of establishing the optimal angle in table 4, give different results. This should not be surprising since the two methods are calculating two different angles. The minimum area method calculates the angle that results in the minimum area as long as the demand is always satisfied. The best fit method gives the angle at which the collected insolation curve follows most closely the demand curve. However, in this case there will be days when the demand is not satisfied. As can be seen from table 4, the minimum area method, in all cases, results in a lower angle of tilt and higher insolation than the best fit method.

It will also be noticed that the minimum area method always gives the same result of 56° for the cases of constant energy demand all year round and in the winter months and constant hot water demand all year round and in the winter months. This is because in all these cases, the minimum radiation will occur on the same day(s). The same happens with the minimum area for the summer months for the constant energy demand and the constant hot water demand.

There is very little difference between the result based on the best fit for the constant energy demand and the constant hot water demand for the summer months. This is because the constant hot water demand is based on the difference between 65° and ambient temperature which varies very little over the four months of summer.

The difference between the results for the constant energy demand and the constant hot water demand, based on the best fit, is more pronounced.

6. Conclusions

A computer model to calculate the irradiation falling on a plane at a given angle of inclination and azimuth was developed successfully for Malta. A factor was introduced to allow for the effect of water vapour, particles and clouds on the amount of radiation transmitted through the atmosphere. The computer program can also calculate the optimal angle of inclination for a given demand curve which is defined by specifying the demand for each day of the year or period in question. Two different methods for determining the optimal angle were developed. In one, the optimal angle is defined as the angle for the minimum panel area that always satisfies the demand. In the other, the optimal angle is the one which gives the best fit between the demand curve and the insolation curve. In this second method, the insolation curve (insolation vs day) follows as closely as possible the demand curve, and results in a lower panel area since there will be days when the insolation is not enough to meet the demand.

7. References

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