

Development of a mathematical analysis model for solar updraft tower plant (SUTP) system

Young Jae Choi¹, Yong Jin Kim¹, Sub Lee Song² and Yoon Won Park¹

¹ Department of Nuclear and Quantum Engineering, Korea Advanced Institute of Science and Technology 291 Daehak-ro, Yuseong-gu, Daejeon, Republic of Korea

² Sustainable Energy and Environment Convergence Education Team, Handong Global University, 558, Handong-ro, Heunghae-eup, Buk-gu, Pohang, Gyeongbuk, Republic of Korea

Abstract

In a solar updraft tower plant (SUTP) system, heated air by solar heat under the transparent collector rises up through the updraft tower. When wind passes by, the turbine generator under the tower produces electricity. New mathematical analysis model was developed with considerations about realistic physical pressure drops through the whole SUTP system. The new analysis model was validated by the experimental data of prototype plant in Manzanares, Spain. The new model was well corresponded with the experimental data.

Keywords: *Solar chimney, Manzanares prototype, SUTP*

1. Introduction

The use of fossil fuel caused significant climate changes like global warming. This situation makes the melt of glacier, sea water level rise. To solve these problems, renewable energy sources are emphasized these days like wind energy and solar energy. The solar updraft tower plant (SUTP) is considered as a promising alternatives among various renewable energy sources. Solar updraft tower system is an electricity generating system using updraft air flow generated by solar energy. The SUTP is composed of three components such as solar collector, updraft tower and power generators. The schematic diagram of SUTP is shown in Fig. 1. In the SUTP system, heated air under the transparent collector induces updraft air flow to the tower due to density difference between inside and outside of the chimney. Air turbine is located at bottom of solar tower to generate electricity using this highly accelerated air. In this system, air under transparent collector is heated up and density difference of air induces updraft air flow in tower located at center of collector. When heated air flow by solar collector rise up through the solar chimney, the turbine generator produce the electricity in the bottom of chimney. The SUTP would be effective in the area of the large amount of solar energy and wide geographical areas for sufficient air flow like the desert regions. Compared to conventional solar power generation, this system has several advantages, such as very simple structure and low maintenance cost.

The first solar chimney design was developed by J. Schlaich, and a pilot plant prototype was built in Manzanares, Spain, in 1981. The 50-kW prototype operated and produced electricity for 8 years, showing the feasibility of a solar chimney power plant. An experimental evaluation of the prototype and

the preliminary test results of the prototype plant were reported by Haff et al. [1, 2]. These authors presented the basic principles of solar chimney power plants, such as the construction cost and large-scale plants. Bernardes et al. [3] developed an analytical and numerical model for a solar chimney power plant, and heat transfer under the collector was investigated using thermal analysis. A water storage system was established under the collector. Additionally, simulation calculations were compared with experimental results from the pilot plant in Manzanares. Pretorius et al. [4, 5] evaluated a numerical model for the performance of a large-scale solar chimney power plant. A computer simulation program was used to solve the governing equation and heat transfer equation. A comprehensive analysis of the solar chimney was conducted under various situations. A preliminary study examining the influence of several factors was conducted using a mathematical model. Two comprehensive studies from the Pretorius and Bernardes papers on heat transfer in the collector were summarized by Bernardes et al. [6]. Their paper compared the comprehensive governing equation for the collector and chimney. Additionally, the assumptions pertaining to the governing equations were compared with respect to continuity, momentum and the energy equation. The convective heat transfer schemes in the collector were compared under several situations. Zhou et al. [7] conducted a comprehensive review of solar chimney power plants, providing information on the description, physical process, experimental and theoretical study status, among others. Koonsrisuk et al. [8] developed a mathematical model of a solar chimney power plant and validated the result with an actual physical plant. They predicted the performance of a large-scale commercial solar chimney and found the optimum design for high power output. Guo et al. [9] predicted the annual performance of an SCPP in Sinkiang, China. The effect of the collector and chimney radii on the power output was estimated, and the potential annual power requirement of SCPPs in the Hami region was presented.

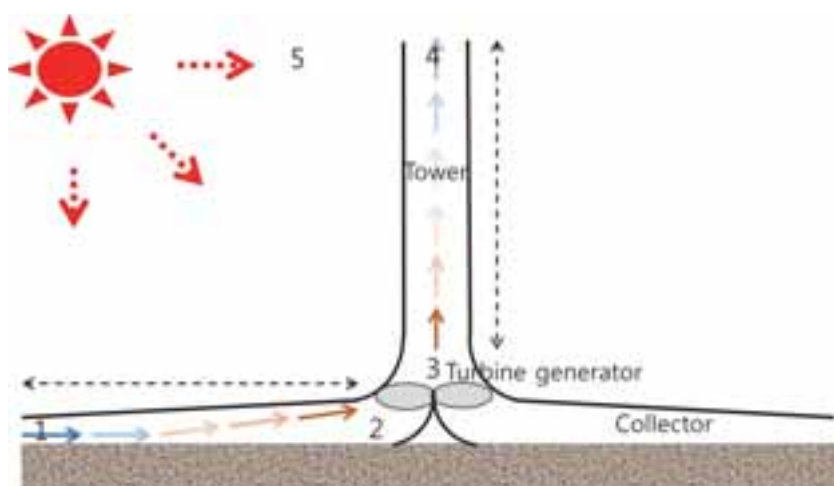


Fig. 1: Schematic diagram of SUTP

In this study, new mathematical analysis model was developed with considerations about realistic physical pressure drops through the whole SUTP system. The new analysis model was validated by the experimental data of prototype plant in Manzanares, Spain. Temperature rise of air through collector was calculated considering heat transfer of collector system, including absorption, reflection, and radiation. Pressure drop and mass flow rate was calculated through iteration process using MATLAB. The final output power can be predicted from pressure, density, temperature and pressure drop at each

point. All the essential parameters, such as temperature, pressure, density, and output power, were compared with the results of Manzanares pilot plant and were found to be in a good agreement.

2. Mathematical model development

2.1 Assumption

The new model is based on several assumptions: ideal gas law, steady state, radially one-dimensional flow under collector, the adiabatic tower wall. From these assumptions, the thermal analysis in the collector was conducted. The solar irradiation was the only heat source. The absorbed heat into the collector roof and the ground was assumed to transfer the heat into the air flow by convection heat transfer. The radiative heat emissions from the collector roof and the ground were also considered. The solar tower scale was selected to dimension of the pilot plant in Manzanares, Spain to validate the measured data with our developed mathematical model. But the roof height of collector was inclined to the tower center. Thus, inlet collector height is 1.2 m and height is increased linearly, so outlet collector height is 2.5 m.

Tab. 1: Dimensions of solar updraft tower plant

Tower height	194.6 m
Tower radius	5.08 m
Collector radius	122 m
Collector inlet height	1.2 m
Collector outlet height	2.5 m

In calculation of turbine pressure drop, the ratio of the pressure drop across the turbine to the total driving pressure was assumed to 2/3. Many researcher have assumed the optimum values of pressure ratio is 2/3. In this study, the pressure ratio across the turbine was optimized using our developed mathematical model.

2.2 Thermal analysis in the collector

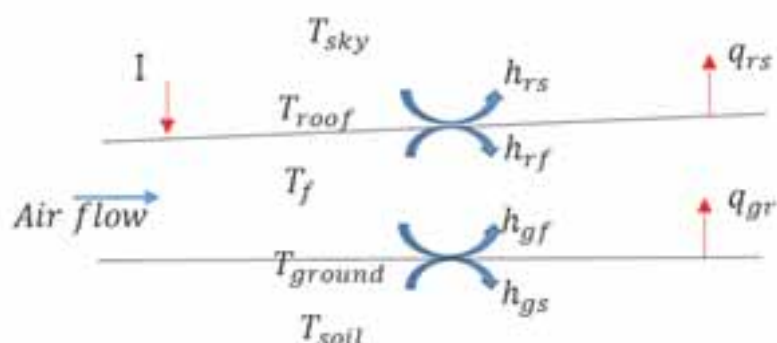


Fig. 2: Thermal analysis in the collector

The schematic diagram of heat transfer circuit in the collector is shown in Fig. 2. h_{rs} , h_{rf} , h_{gf} and h_{gs} is convective heat transfer coefficient and q_{rs} , q_{gr} is radiative heat flux from roof to sky and from ground to roof. The solar irradiation heated the collector roof and ground, then the heated roof and ground transfer the heat the air flow in the collector. The heat transfer equation is following:

$$\text{Roof: } I\alpha_r + q_{gr} = h_{rs}(T_r - T_s) + h_{rf}(T_r - T_f) + q_{rs} \quad (\text{eq. 1})$$

$$\text{Air: } h_{rf}(T_r - T_f) + h_{gf}(T_g - T_f) = \frac{\dot{m}c_p}{2\pi r} \frac{dT_f}{dr} \quad (\text{eq. 2})$$

$$\text{Ground: } I\tau_r\alpha_g = h_{gf}(T_g - T_f) + h_{gs}(T_g - T_{soil}) + q_{gr} \quad (\text{eq. 3})$$

Left side of equation is incoming heat from the surrounding and right side of equation is output to surrounding. Equation (1),(2),(3) is thermal heat transfer equilibrium equation in roof, air and ground. I is the total solar irradiation on collector, τ_r is transmittance of roof and α_r and α_g are the absorption coefficient of the roof and ground. Temperature of sky is calculated by the ambient temperature. The temperature of soil was assumed constant at all area. The equation of radiative heat flux and heat transfer coefficient is following:

$$q_{gr} = \frac{1}{\frac{1}{\varepsilon_g} + \frac{1}{\varepsilon_r} - 1} \sigma (T_g^4 - T_r^4) \quad (\text{eq. 4})$$

$$q_{rs} = \varepsilon_r \sigma (T_r^4 - T_{sky}^4) \quad (\text{eq. 5})$$

$$h_{rs} = 3.87 + 0.0022 \left(\frac{v_w \rho c_p}{Pr^{\frac{2}{3}}} \right), \quad (v_w: \text{ambient air velocity}) \quad (\text{eq. 6})$$

$$h_{rf} = h_{gf} = 3.87 + 0.0022 \left(\frac{v \rho c_p}{Pr^{\frac{2}{3}}} \right), \quad (v: \text{collector air velocity}) \quad (\text{eq. 7})$$

$$h_{gs} = 2 \sqrt{\frac{k \rho c_p}{\pi t}} \quad (\text{eq. 8})$$

When the collector roof temperature exceeds the ambient temperature and collector inside situation, equation (6) and (7) can be used. Equation (8) is ground heat transfer coefficient and t is second of time.

2.3 Pressure drop

The driving force for air flow in the SUTP system was designated by considering the density difference between inlet and outlet of the tower. It is described as (eq. 9). The pressure drops due to friction losses through the overall SUTP system would disturb the flow. Along the flow path of the SUTP system, friction losses and form losses on the tower were considered. The ratio of turbine pressure drop to total driving force was assumed to be 0.66. It is an optimized value for high power performance of turbine. Mass flow rate of the system was calculated by computing the iterative in-house MATLAB code.

$$\text{Driving force} = \Delta P_{tot} = \int_0^{H_c} (\rho_{out} - \rho_{in}) g dz \quad (\text{eq.9})$$

$$\Delta P_{tur} = \Delta P_{tot} - \Delta P_{drop} \quad (\text{eq.10})$$

$$\Delta P_{drop} = \Delta P_{tur.in} + \Delta P_{f,chim} + \Delta P_{k,out} \quad (\text{eq.11})$$

$$\Delta P_{tur.in} = K * \frac{\rho}{2} * V_2^2 : \text{Turbine inlet form pressure drop}$$

$$\Delta P_{f,chim} = f * \frac{H}{D} * \frac{\rho}{2} * V^2 : \text{Chimney frictional pressure drop}$$

$$\Delta P_{k,A} = K_{out} * \frac{\rho_A}{2} * V_4^2 : \text{Chimney outlet form pressure drop}$$

The pressure drop of turbine makes the wheel rotate, generating the electricity. It is the difference between the driving force and pressure drop over chimney as (eq.10). The pressure drop of system is the sum of the pressure drop over a form loss at turbine inlet, friction loss at chimney wall and chimney outlet form loss as (eq. 11). The ultimate goal of the draught equation is to determine the pressure drop across the turbine. Using this pressure drop, the power generated by the turbine may be calculated.

2.4 Power output

The flow of air through the tower of the solar tower power plant drives a turbine at the base of the tower. The turbine drives a generator which generates electricity. The power output of the system was calculated by the (eq. 12). η_{tg} is turbine efficiency and V_{avg} is volumetric flow rate of air. For the maximum of power on system, the pressure ratio across the turbine play an important role in calculating the performance. Many researcher have assumed the optimum values of pressure ratio is 2/3. In this study, the pressure ratio across the turbine was optimized using our developed mathematical model.

$$P_{out} = \eta_{tg} \Delta P_{tur} V_{avg} = \eta_{tg} \Delta P_{tur} v_{tur} A_c \quad (\text{eq.12})$$

2.5 Calculation algorithm

The overall power performance of solar updraft tower plant was calculated following an algorithm using MATLAB. The procedure of calculation is following figure 3. The pressure, temperature and density in region of 2, 3 and 4 were obtained using the thermal analysis and several equation. Then, the mass flow rate of system was assumed to some values. The air velocity of each region can be calculated using mass conservation equation. Air flow pressure drop of system can be calculated at several regions. The form pressure drop at chimney inlet, friction pressure loss at chimney wall and outlet form pressure loss at chimney outlet were obtained, then the velocity can be calculated using the pressure drop conservation equation. Mass flow rate was recalculated from obtained velocity. Then, the calculation cycle was recurred until the small difference between the previous value and current value. After the iteration process, the power output of system can be calculated.

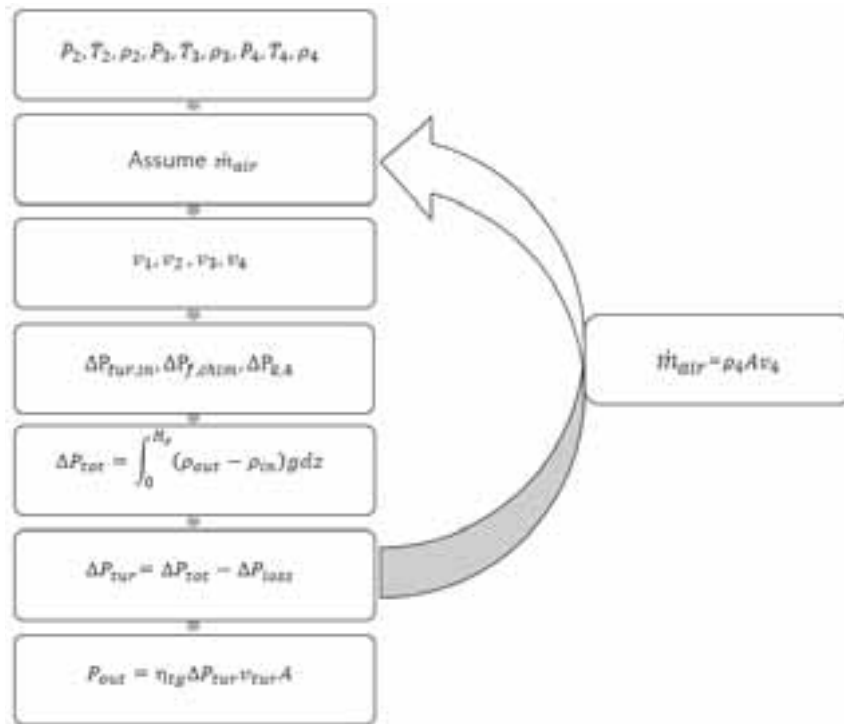


Fig. 3: Calculation algorithm of SUTP

3. Results and Discussion

3.1 Validation of developed model with manzanares experimental data

To validate the new mathematical model, information of the ambient temperature and solar irradiation data on 2nd of September in 1982 from the Haff's paper were utilized for the initial condition. The calculated result of the power output, pressure drop, velocity and temperature change in the collector were compared with the real data. In fig. 4, solar irradiation at Manzanares region was measured and the profile was obtained at all day long. Ambient temperature at Manzanares region was also figured in fig. 5.

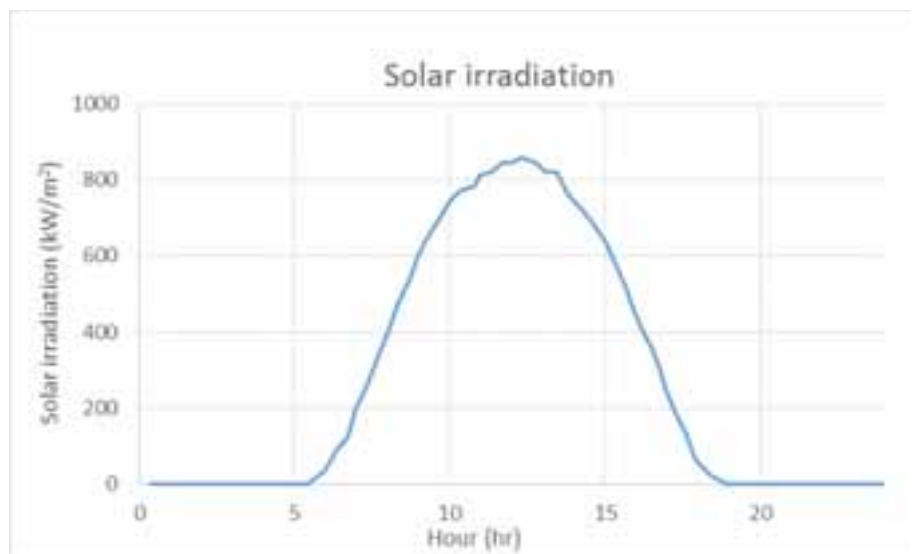


Fig. 4: Measured solar irradiation at Manzanares on 2 September 1982

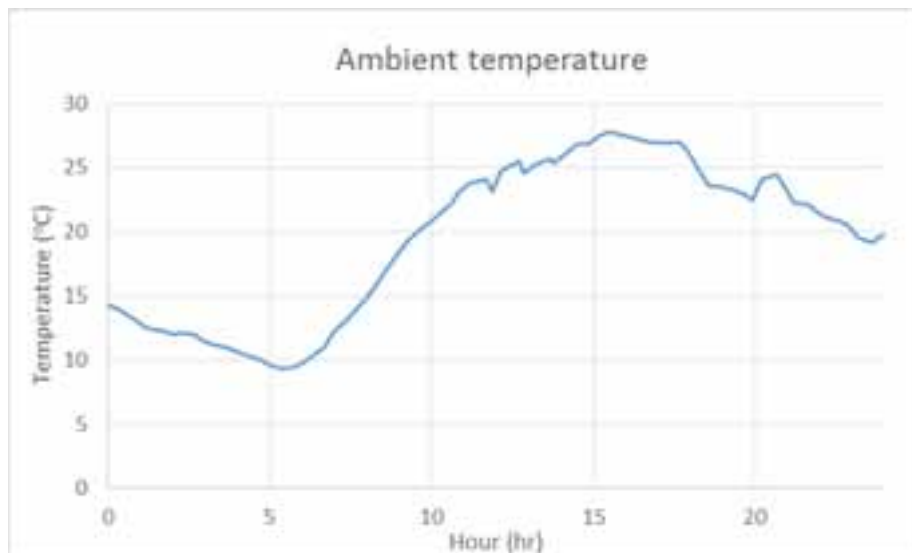


Fig. 5: Measured ambient temperature at Manzanares on 2 September 1982

In calculation of SUTP performance, ambient temperature and solar irradiation was used as initial conditions. Using this values, the calculated output power was compared with measured values at fig. 6. Overall trend of power is similar, but the calculated power is slightly higher at early morning. The maximum power is about 34 kW at the 12 pm. The power at night is zero because there is no solar irradiation. At fig.7, temperature rise in collector was compared with measured data. The trend is similar, but the temperature rise until 8 am is quite different. But the temperature rise at daytime is similar from about 15 °C to 20 °C. At fig. 8, air velocity at chimney was compared with the measured data. Measured velocity profile is quite fluctuating, so it is difficult to compare each other. But, overall trend of velocity is similar, and the calculated velocity data is underestimated than measured one. At fig. 9 and 10, the calculated pressure difference at total system and turbine system were compared with measured one. The trends are also similar each other.

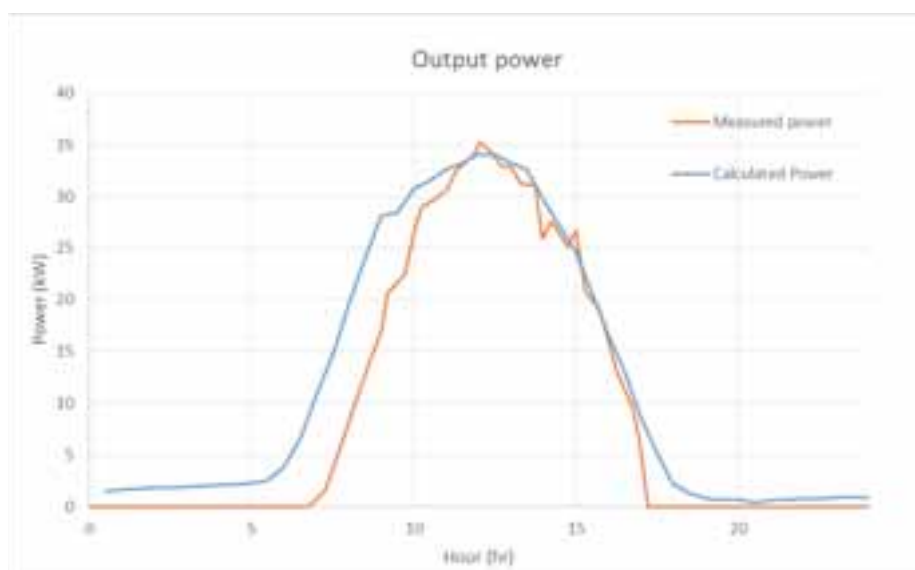


Fig. 6: Comparison of calculated power with measured power

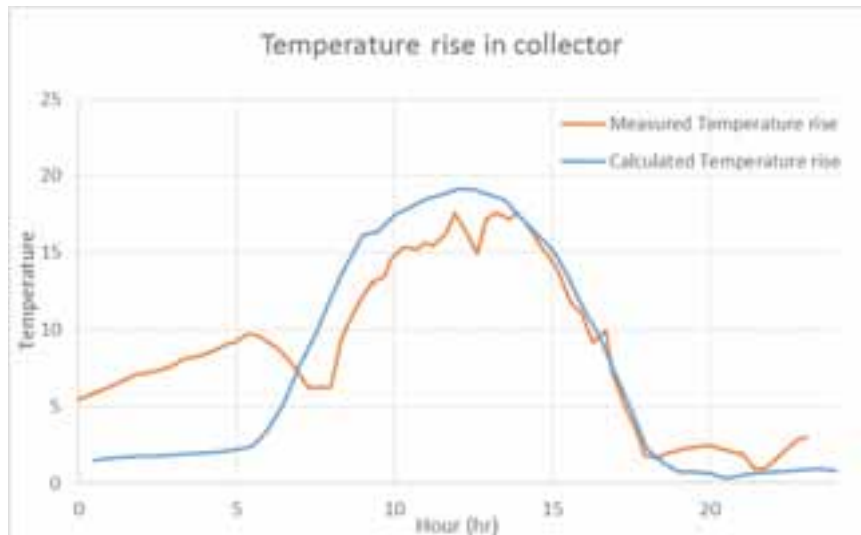


Fig. 7: Comparison of calculated temperature rise with measured temperature rise at collector

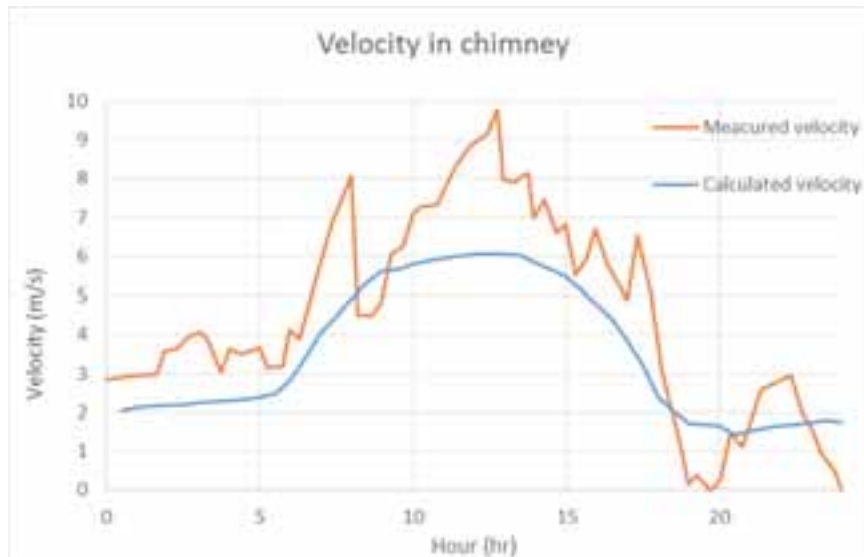


Fig. 8: Comparison of calculated velocity with measured velocity at chimney

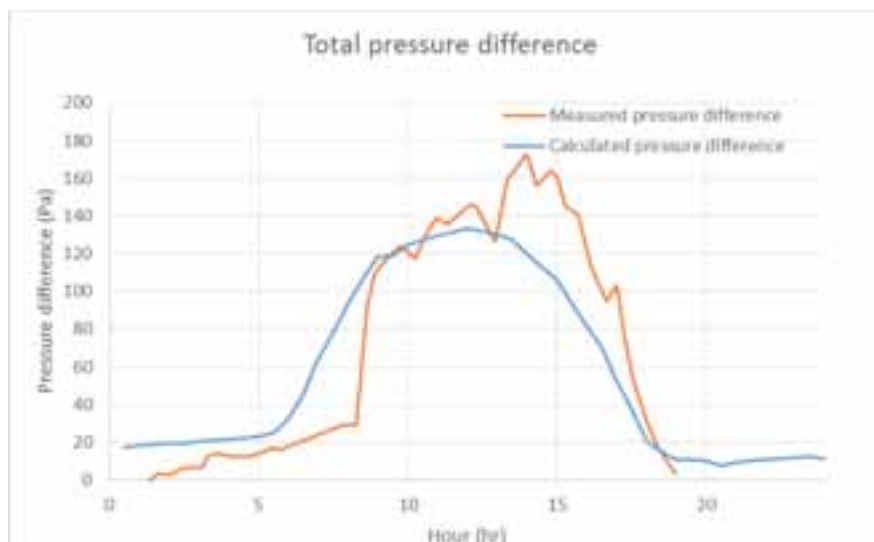


Fig. 9: Comparison of calculated total pressure difference with measured total pressure difference

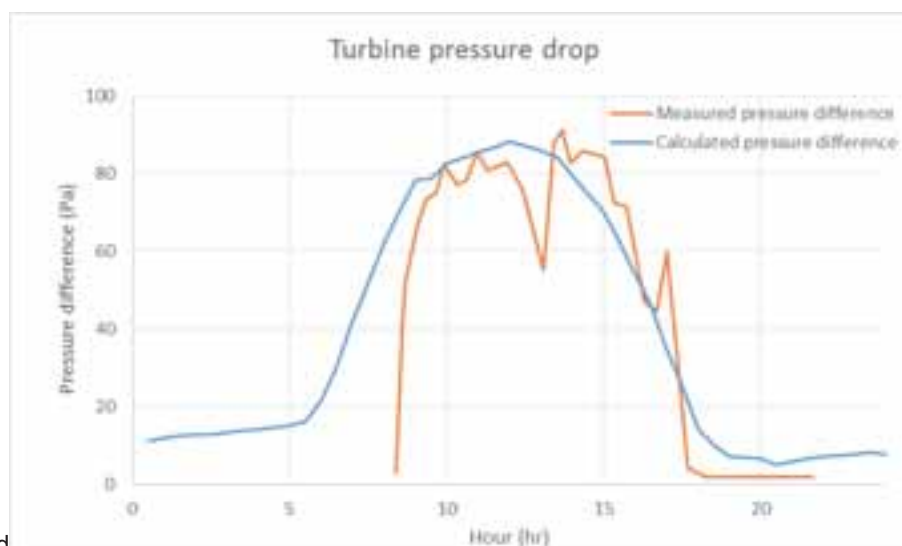


Fig. 10: Comparison of calculated turbine pressure difference with measured turbine pressure difference

3.2 Large scaled SUTP

Idea of solar updraft tower plant was suggested for the high electricity production at the dry area where the solar energy is abundant and water storage is deficient. For the high electricity production, scale of solar chimney should be expanded to obtain the high power. Many research evaluate performance of large scaled solar chimney. In this paper, the large scaled tower was assumed to operate ordinarily at same condition of Manzanares initial value like ambient and solar irradiation. At that conditions, the size of solar tower was changed like following table. 2. Slope of collector is linearly tilted from inlet to outlet of roof. Other basic values are same with values of the manzanares calculation. From the figure 11 to 15, the result of the large scale tower plant was obtained. The maximum power is about 130 MWe at 12 pm. The temperature rise in the collector is maximum 35 °C at 12 pm. The air flow velocity at chimney is highest value of 20 m/s at day time. The maximum pressure difference profile followed also similar trend. From the large scaled analysis, we could estimate the performance of system at the specific tower dimension.

Tab. 2: Dimensions of large scaled solar updraft tower plant

Tower height	1,000 m
Tower radius	100 m
Collector radius	3,000 m
Collector inlet height	5 m
Collector outlet height	10 m

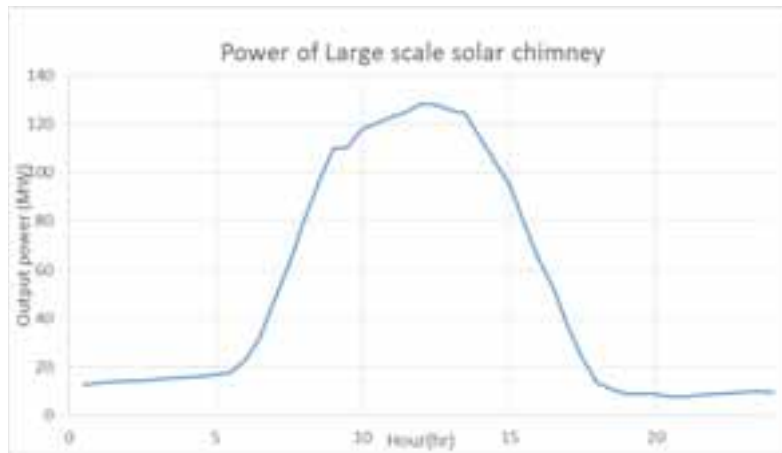


Fig. 11: Calculated output power of large scaled SUTP

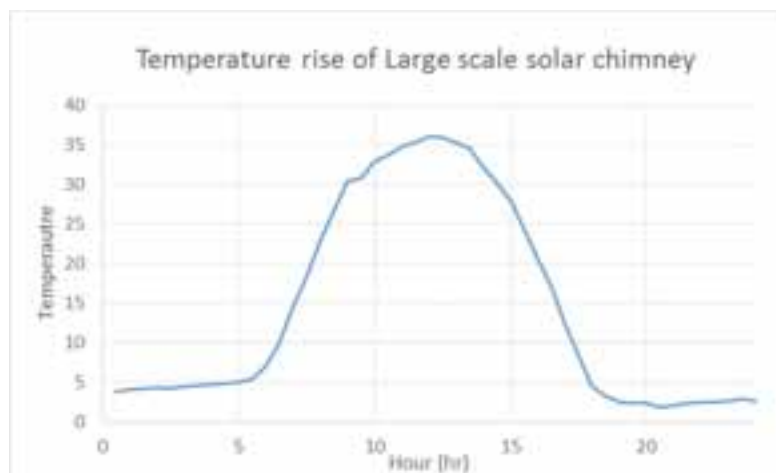


Fig. 12: Calculated temperature rise of large scaled SUTP

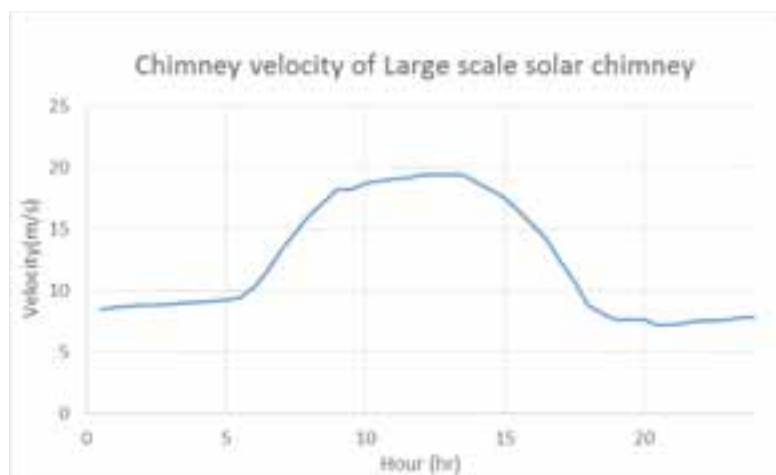


Fig. 13: Calculated chimney velocity of large scaled SUTP

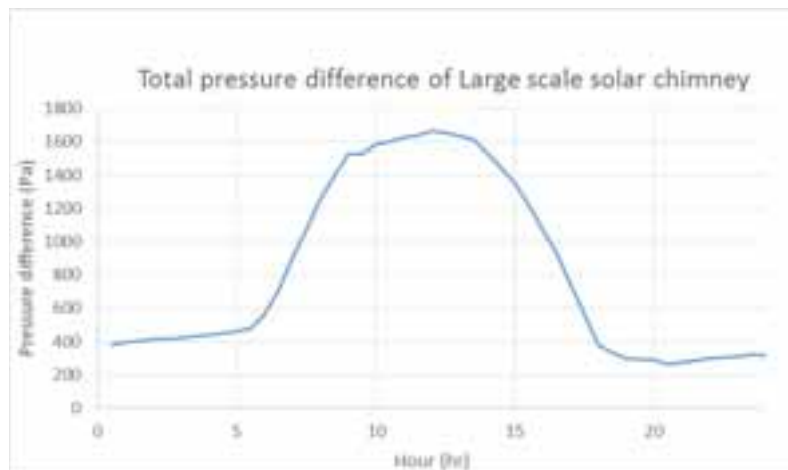


Fig. 14: Calculated total pressure difference of large scaled SUTP

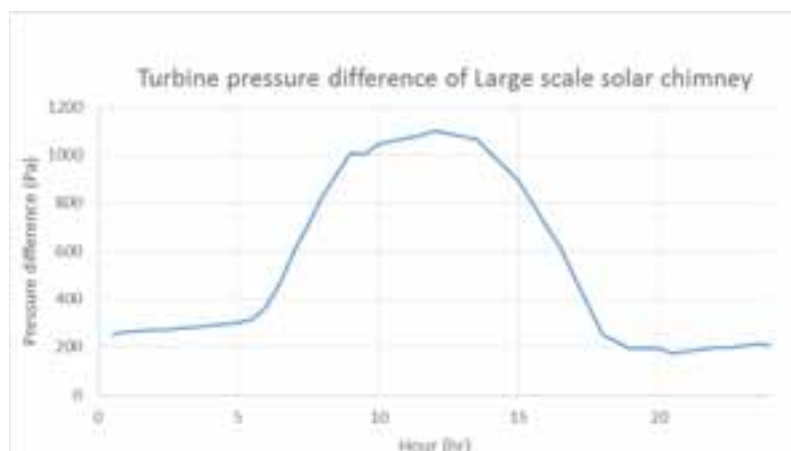


Fig. 15: Calculated turbine pressure difference of large scaled SUTP

4. Conclusions

A new mathematical analysis model was developed to evaluate the performance of SUTP system. The prediction of new model was compared with experimental data. The new model was validated with measured data, and the trends of overall data was considerably well-estimated. By using this mathematical method, large scaled SUTP system was also evaluated. The output power scale was about 100MWe. But there are many defect on our model development, so more comprehensive study for improving the model should be conducted.

5. Reference

- [1] Haaf W, Friedrich K, Mayr G, Schlaich J. Solar chimneys Part I: principle and construction of the pilot plant in Manzanares. *Int J Sustain Energy* 1983;2(1):3–20.
- [2] Haaf W. Solar chimneys Part II: preliminary test results from the Manzanares pilot plant. *Int J Sol Energy* 1984;2(2):141–61.
- [3] Bernardes MAd, Voß A, Weinrebe G. Thermal and technical analyses of solar chimneys. *Sol Energy* 2003;75(6):511–24.

- [4] Pretorius, J. P., 2004, "Solar Tower Power Plant Performance Characteristics," M.Sc.Eng. thesis, University of Stellenbosch, Stellenbosch, South Africa.
- [5] Pretorius, J. P. and Kroger D. G., "Critical evaluation of solar chimney power plant performance," *Solar Energy*, vol. 80, no. 5, pp. 535–544, 2006.
- [6] Bernardes, M.A.d.S., Von Backström, T.W., Kroger, D.G., 2009. Analysis of some available heat transfer coefficients applicable to solar chimney power plant collectors. *Solar Energy* 83, 264–275.
- [7] Zhou XP, Wang F, Ochieng RM. A review of solar chimney power technology. *Renew Sustain Energy Rev* 2010;14(8):2315–38.
- [8] A. Koonsrisuk, T. Chitsomboon, Mathematical modeling of solar chimney power plants, *Energy*, 51 (2013), pp. 314–322
- [9] P. Guo, J. Li, Y. Wang, Annual performance analysis of the solar chimney power plant in Sinkiang, China, *Energy Convers Manage*, 87 (2014), pp. 392–399