# Mathematical Modeling of a Nano-Engineered Photovoltaic/Thermal (PV/T) System

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# Abstract

Integrating photovoltaic (PV) cells with the solar thermal collector is found beneficial and attractive in the context of the simultaneous production of electricity and heat. Using nanofluid and air as the coolants, a transient mathematical model of a bi-fluid photovoltaic/thermal (PV/T) is developed. Experimental validation of the mathematical model is performed using results from the published work. For nanofluid, aluminum oxide  $(Al_2O_3)$  nanoparticles with different concentrations of 0.15 wt%, 0.3 wt%, 0.45 wt%, and 0.6 wt% were dispersed in a base fluid (pure water). The interdependence transient temperature responses of the PV/T collector components are simulated using MATLAB® software. The overall collector performance is predicted and compared when both fluids are to be operated independently and simultaneously. The simulation results indicate that when the fluids are operated simultaneously, the collector performance is better than the independent mode. It is observed that nanofluid & air based PV/T collector provides a higher performance in comparison with conventional heat exchanger systems used in this study.

Keywords: Mathematical model, Bi-fluid PV/T, Nano-engineered, Model validation

## 1. Introduction

A photovoltaic/thermal (PV/T) system possesses better solar harvesting ability in comparison with an individual solar thermal collector or Photovoltaic (PV) system (Hussain and Lee, 2015). Therefore, the PV/T collectors can generate more energy per unit surface area than that of conventional collectors. In a hybrid-PV system, the PV cells performance can be further improved by circulating appropriate heat transfer fluid across its surface. By considering this fact various types of heat transfer fluids having different thermophysical properties have been studied for the PV/T systems (Bhattarai et al., 2012; Rejeb et al., 2016). Water and air among them were the most frequently used cooling fluids (Imtiaz Hussain et al., 2015; Kim et al., 2014). Furthermore, the literature review shows that the rate of heat transfer between the absorber and circulating fluid can be enhanced by increasing the thermal conductivity of the base fluid. The thermal conductivity of the coolant is improved by dispersing nano-sized metallic, metal oxides and nanotube particles in the base fluid. Therefore, application of nanofluid as a coolant for the PV/T system seems to be more promising.

Since the mid-1970s, when the research on the PV/T system started many researchers focused their efforts to improve its performance to get maximum possible thermal and overall efficiency. Chow (2003) proposed an explicit dynamic model that can predict the PV module and circulating fluid temperatures even under rapidly fluctuating solar radiation. Rejeb et al. (2016) evaluated the effect of nanofluid as a heat transfer fluid on the performance of the PV/T system. Among various colloidal solutions, the suspension of copper nanoparticles in pure water provided higher overall energy efficiency compared to alumina. Simultaneous application of two fluids as the heat transfer fluids for the PV/T system offer a great range of benefits in terms of a higher thermal and electrical output. As mentioned by Abu Bakar et al. (2014), the dual fluids based PV/T system was first built and tested by Tripanagnostopoulos (2007). For the purpose of heat extraction improvement, the PV module was integrated with water or air heat extraction elements. Thenceforward, further studies on the effectiveness of the bi-fluid PV/T design was carried out by applying low cost modifications such as introducing serpentine-shaped copper tube as the water heat extraction element transverse to the air flow and fins in the air channel parallel to the air flow direction (Abu Bakar et al., 2014; Jarimi et al., 2016). According to published literature, no previous study using nanofluid and air as the heat extraction fluids for PV/T system has been reported.

This study suggests a transient mathematical model of a bi-fluid PV/T system with the novelty lies in design and the heat extraction components in which nanofluid along with air is introduced. This research seeks to overcome the challenges and gaps to develop a bi-fluid based PV/T system which can significantly decrease PV cells temperature and increase the overall performance of the system.

# 2. Design concept

A novel bi-fluid PV/T system mainly consisted of standard off the shelf mono-crystalline PV module, copper pipes as the nanofluid heating component, and a single pass air duct as the air heating component. Schematic diagram of the nano-engineered bi-fluid PV/T system is shown in Fig. 1. The flow paths for the nanofluid and air are designed in such a way that there is no physical interaction between two fluids. However, interdependence heat exchange occurs across both heat exchanger components. In order to improve heat transfer rate to the flowing air, the surfaces of the nanofluid carrier pipe and the PV panel are taken as matt black. To heat transfer improvement, a set of baffles is introduced transverse to the direction of airflow. The parallel-arranged fluid carrier pipes are attached directly to the PV module back surface instead of using absorber plate. This direct coupling of PV module with nanofluid heating component results in a decrease of thermal resistance and an increase of heat transfer rate between the PV surface and circulating fluids.





Fig. 1: (a) Three dimensional view (b) Cross-section view of the bi-fluid PV/T system.

Fab. 1: Parameters for simulation
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Description		Value	Description	Value	
PV module (length & width)	L & W	1.62 m & 0.98 m	Pipe Density	$ ho_t$	2702 kg/m <sup>3</sup>
PV absorptivity	$\alpha_p$	0.9	No. of tubes	-	9
PV emissivity	$\varepsilon_p$	0.88	Pipe spacing	-	0.11m
PV specific heat	$C_p$	900 J/kgK	Back panel Density	$ ho_b$	$20 \text{ kg/m}^3$
PV reference efficiency (%)	ŋ <sub>r</sub>	17.3	- Panel Specific heat	$C_b$	670 J/kg K
Packing factor	Р	-	- Panel Thermal conductivity	$K_b$	0.034 W/m K
Temperature coefficient	$\beta_r$	0.0045/°C	Aluminum oxide	$Al_2O_3$	-

Pipe inner diameter	$D_i$	0.008 m	Al <sub>2</sub> O <sub>3</sub> Thermal conductivity	-	30  W/m K
Pipe thickness	$\delta_t$	0.0012 m	Al <sub>2</sub> O <sub>3</sub> Specific heat	-	773 J/kg K
Pipe Specific heat	$C_t$	903 J/kgK	Al <sub>2</sub> O <sub>3</sub> Density	-	3890 kg/m <sup>3</sup>

# 3. Mathematical model

A model based on conservation of energy of bi-fluid PV/T system components is developed and analyzed in transient conditions using ODE solver in MATLAB®. Details of simulation parameters and thermo-physical properties are presented in Table 1. To evaluate the electrical and thermal performances of a bi-fluid PV/T system, heat balance equations for different components such as PV module, copper pipes, nanofluid, circulating air, and back panel are presented as follows:

#### For PV

$$\begin{split} M_p C_p (dT_p/dt) &= G\alpha_p - E - h_{wind} A_{p\infty} (T_p - T_{\infty}) - h_{p\infty} A_{p\infty} (T_p - T_{\infty}) - h_{pt} A_{pt} (T_p - T_t) - A_{pa} h_{pa} (T_p - T_a) - h_{pb} A_{pb} (T_p - T_b) \end{split}$$

$$\begin{aligned} & (eq. 1) \\ E &= GP\eta_e \end{aligned}$$

For copper pipe

 $M_{t}C_{t}(dT_{t}/dt) = h_{pt}A_{pt}(T_{p} - T_{t}) - A_{tn}h_{tn}(T_{t} - T_{n}) - A_{ta}h_{ta}(T_{t} - T_{a}) - h_{tb}A_{tb}(T_{t} - T_{b})$ (eq. 3)

For nanofluid in pipe

$$M_{n}C_{n}(dT_{n}/dt) = \dot{m}_{n}C_{n}(T_{n,o} - T_{n,in}) + A_{tn}h_{tn}(T_{t} - T_{n})$$
(eq. 4)

For inside air

$$M_{a}C_{a}(dT_{a}/dt) = \dot{m}_{a}C_{a}(T_{a,o} - T_{a,in}) + A_{pa}h_{pa}(T_{p} - T_{a}) + A_{ta}h_{ta}(T_{t} - T_{a}) + h_{ab}A_{ab}(T_{a} - T_{b})(eq. 5)$$

For back panel

$$M_{b}C_{b}(\partial T_{b}/\partial t) = h_{tb}A_{tb}(T_{t} - T_{b}) + h_{pb}A_{pb}(T_{p} - T_{b}) + h_{ab}A_{ab}(T_{a} - T_{b}) - h_{b\infty}A_{b\infty}(T_{b} - T_{\infty})$$
(eq. 6)

where, G,  $\alpha_p$ , E, and P are the solar radiation, the absorptivity of PV cells, electrical power and packing factor, respectively. M, C, T, A, m, and h are the mass, specific heat, temperature, surface area, mass flow rate and heat transfer coefficient, respectively. The subscripts p, t, n, a, b, and  $\infty$  are denoted the PV, copper pipe, nanofluid, inside air, back panel insulation and ambient air, respectively. The subscripts p $\infty$ , pt, pa, tn, ta, ab, tb, pb, and b $\infty$  are denoted the heat transfer contacts between the PV and ambient air, PV and pipe, PV and inside air, pipe and nanofluid, pipe and inside air, back panel and inside air, back panel and pipe, back panel and PV, and back panel and ambient air, respectively. The subscripts in and o are denoted fluid inlet and outlet, respectively. The electrical, thermal and primary energy saving efficiencies of dual-fluid PV/T systems are calculated using following expressions (Baljit et al., 2017):

$$\eta_e = \eta_r [1 - \beta_r (T_p - T_r)]$$
(eq. 7)

$$\eta_{\rm th} = \frac{\dot{m}_{\rm n} C_{\rm n} \left( T_{\rm n,o} - T_{\rm n,in} \right) + \dot{m}_{\rm a} C_{\rm a} \left( T_{\rm a,o} - T_{\rm a,in} \right)}{A_{\rm c} G} \tag{eq. 8}$$

$$\eta_{\rm PVT} = \eta_{\rm th} + \eta_e / \eta_{\rm pp} \tag{eq. 9}$$

where,  $\beta_r$  and  $A_c$  are the solar cells temperature coefficient and collector area, respectively.  $\eta_r$  is the efficiency of the solar cells at a reference temperature  $(T_r)$ .  $\eta_e$ ,  $\eta_{th}$  and  $\eta_{PVT}$  are the electrical, thermal and primary energy saving efficiencies, respectively.  $\eta_{pp}$  is the power generation efficiency of the conventional power plant.

# 4. Results and discussions

#### 4.1. Model validation

Model validation of a proposed PV/T system is performed in two steps, at first the nominal operating cell

temperature (NOCT) of the PV module is predicted using model and then validated against NOCT value obtained from manufacturer's datasheet e.g. 47 °C (PV-MJU240GB). The percentage difference between predicted and datasheet value is less than 1.8%. Second step is a validation of thermal component of air heating against the experimental data (Table 2) for unglazed PV/T air heating system presented by Joshi et al. (2009). To ease the comparison, the nanofluid flow rate is set to be zero. During simulation, similar design and environmental conditions are considered, as presented in (Joshi et al., 2009), other information which is not given in this research were taken from the study by Abu Bakar et al. (2014).

Time (hour)		8	9	10	11	12	13	14	15	16	17
PV cells	Exp	37.60	41.40	47.90	50.40	54.90	54.70	52.90	50.70	47.20	42.30
temperature (°C)	Sim	38.01	41.65	48.52	50.85	55.70	55.50	53.58	51.32	47.79	42.66
Outlet air	Exp	33.20	36.30	38.90	41.70	46.10	41.50	46.40	44.50	43.00	40.30
temperature (°C)	Sim	33.59	36.71	39.33	42.21	46.69	42.11	46.91	44.12	43.49	40.77

Tab. 2: Simulated values by model against experimentally measured data by (Joshi et al., 2009)

Using validated model, the overall performance of a bi-fluid PV/T system is evaluated under different modes of fluid operation. During independent mode, one of the heat extraction fluids is kept stagnant, while other fluid is operated at a designated flow rate. For the simultaneous mode of fluids operation, both fluids are operated at the same time. During simulation, all heat transfer coefficients were calculated in real time.

#### 4.2. Simulation results

To locate the best concentration, different weight fractions of  $Al_2O_3$  nanoparticles in a base fluid have been considered. Generally, a variation of nanoparticles concentration affects directly the thermo-physical properties (thermal conductivity and specific heat in particular) of the colloidal solution, and hence the heat transfer rate. Variation of daily solar radiation and ambient temperature are shown in Fig. 2. Influence of  $Al_2O_3$  nanoparticles concentration on the thermal conductivity ratio and specific heat ratio is shown in Fig. 3. Increasing the nanoparticles (np) concentration in the base fluid (bf) increases the thermal conductivity and decreases the specific heat of the  $Al_2O_3$  nanofluid. It is observed that the thermal conductivity ratio (knp/kbf)) increased to 1.162 and the specific heat ratio (Cnp/Cbf) decreased to 0.954 when the  $Al_2O_3$  weight fraction increased from 0 to 0.75 (wt%). This can be attributed to the higher thermal conductivity and lower specific heat of the  $Al_2O_3$ nanoparticles compared to water as the base fluid.



Fig. 2: Daily variation of (a) solar radiation (b) ambient temperature

The PV cells temperature has been predicted using different working fluids both simultaneously and independently. Fig. 4 depicts the PV cells temperature under different modes of fluid operation namely:  $Al_2O_3/air$ , water/air, nanofluid, air, and without the heat transfer fluid. Compared to the PV temperature obtained without working fluid (69.4 °C), the average PV temperature drop by 7.8, 9.2, 11.3, and 13.6 °C when either one or two fluids are operated such as air,  $Al_2O_3$  nanofluid, water/air, and  $Al_2O_3/air$ , respectively. It is worth to note that introducing bi-fluid heat extraction component, in particular,  $Al_2O_3/air$ , underneath the PV module increases not only its electrical efficiency but also produces high-temperature heat at the same time.



Fig. 3: Variation of thermal conductivity ratio and specific heat ratio of Al<sub>2</sub>O<sub>3</sub> with concentration (wt%).



Fig. 4: Predicted PV temperature using different coolant fluids and modes of fluid operations.

The effect of different modes of fluid operation on the daily total efficiency of a PV/T system is observed by varying nanofluid or water flow rate at a fixed air flow rate of 0.05 kg/s (Fig. 5). The total efficiency of the bifluid PV/T system is considerably higher than the system with a single heating component. The simulation results show that the overall energy efficiency of a  $Al_2O_3/air$  based PVT was found significantly higher than that of the cases with water/air and nanofluid heat exchanger systems, owing to additional thermal and electrical energy produced. The use of nanofluid as a heat extraction component for PV/T system provides excellent overall energy performance in comparison with either water or air.



Fig. 5: Total PV/T efficiency with varying nanofluid or water flow rate at fixed air flow rate of 0.05 kg/s.



Fig. 6: PV/T thermal efficiency (a) with different concentration of Al<sub>2</sub>O<sub>3</sub> at stagnant air (b) with varying air flow rate at stagnant nanofluid.

Even though the model is developed considering dual-fluid heat exchangers for the dual-fluid PV/T system. However, by setting one of the fluids in stagnant mode, the given model can also be used for a single fluid operation. Fig. 6 compares the thermal efficiency of the PV/T system with  $Al_2O_3$  nanofluid and air at zero reduced temperature. In addition, the influence of nanoparticle concentrations on the collector performance is also investigated, as shown in Fig. 6a. In this paper, four nanofluid concentrations (0.15 wt%, 0.3 wt%, 0.45 wt%, 0.6 wt%) were selected to investigate. It is observed that the zero-loss thermal efficiencies of the PV/T system with  $Al_2O_3$  nanofluid and air were found to be 65.3% and 34.5%, respectively. Furthermore, the thermal

efficiency of the PV/T system increases with increasing the weight fraction of  $Al_2O_3$  nanoparticles in the base fluid. It should be noted that changing the nanoparticles concentration from 0.15 to 0.6 wt% results in a thermal efficiency enhancement from 58% to 65.3%, respectively. The higher thermal efficiency of PV/T system with nanofluid as a coolant can be explained by an improvement of thermal conductivity of the base fluid by dispersing nanoparticles (Sharma et al., 2017). Thus, enhancement of heat transfer coefficient between the absorber and circulating fluid is observed.

In order to evaluate the overall performance of a nano-engineered dual-fluid PV/T system, it is important to investigate the effects of the simultaneous fluid operation on primary energy saving efficiency. Influence of variable mass flow rate of  $Al_2O_3$  nanofluid (at fixed air flow rate) and air (at a fixed nanofluid flow rate) on the primary energy saving efficiency is shown in Fig. 7. Increasing the nanofluid flow rate from 0.005 to 0.03 kg/s with fixed air flow rate at 0.015 kg/s, 0.035 kg/s, and 0.055 kg/s the maximum primary energy saving efficiency were 88.6%, 89.3% and 90.2%, respectively. Whereas, increasing air flow rate from 0.02 to 0.12 kg/s with fixed nanofluid flow rates at 0.008 kg/s, 0.013 kg/s, and 0.018 kg/s these values were found to be 78.9% to 79.3% and 79.8%, respectively. It is important to note that the percentage increase in the primary energy saving efficiency with the variable nanofluid flow rate is considerably higher than the variable air flow rate. This can be explained by the fact that nanofluid has higher average thermal conductivity than water and air. Therefore, the solar heat being sufficiently absorbed from the PV panel by the circulating nanofluid when the flow rate of air is kept constant.



Fig. 7: Primary energy saving efficiency (a) variable nanofluid flow rate at fixed air flow rate (b) variable air flow rate at fixed nanofluid flow rate.

## 5. Conclusion

The transient numerical analysis of a bi-fluid PV/T system which incorporates the simultaneous application of  $Al_2O_3$  nanofluid and air heat extraction components is performed. The developed transient model can be used to predict the PV/T system's performance for both independent and simultaneous modes of fluid operation. All heat transfer coefficients were calculated in real time during the simulation. The heat transfer rate to the flowing air is improved by introducing a set of baffles transverse to the air flow. The model of a nano-engineered dual-fluid PV/T system is developed and validated against the experimental data given in the literature. The use of  $Al_2O_3$  nanofluid plus air as the dual heat exchanger system provides excellent overall energy performance for the PV/T system in comparison with water plus air and other conventional heat transfer fluids. The developed transient mathematical model can able to accurately predict and optimize the collector performance under a

wide range of varied operating and climatic conditions.

## 6. Acknowledgements

This work was supported by Korea Research Fellowship Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science and ICT (2016H1D3A1938222); International Cooperation grant (No. 20148520011270) from the Korea Institute of Energy Technology Evaluation and Planning (KETEP), funded by the Ministry of Trade, Industry and Energy of the Korean government.

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