

Mathematical modelling of an Innovative Ice Storage System

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Abstract

Ice storage systems are important applications for heating and cooling due to its ability to efficiently store thermal energy for later use, reducing reliance on conventional energy sources during peak demand periods. In combination with heat pumps and solar collectors, ice storages present a large advantage in comparison with other conventional heating and cooling systems. In this work, the mathematical modelling of an innovative ice storage system will be presented. The energy storage in this system, happens in innovative spiral flat registers created by the company ECOTHERM, where internal and external melting can be used as required. The mathematical model will then be further integrated in an overall system and a yearly simulation of its performance will be carried out. In the scope of this study, just the heating performance of the ice storage system were analyzed, while the combined use of the ice storage for heating and cooling will be carried out in the subsequent phases of the project. The results focus on the State of Charge (SOC) and the temperature of the ice storage during the simulated year as well as the energy accumulated in the same period. Moreover, the seasonal performance factor (SPF) of the whole system is also presented. The results obtained with the simulations, were later compared to literature data and present a fair equivalence.

Keywords: ice storage, heat storage, mathematical modelling, state of charge

1. Introduction

Substantial progress has been achieved in the last decade in the application of renewable energies, from different sources, to improve energy-storage technologies with the objective of balancing energy supply and demand (Gan et al. 2020). Phase change thermal storage systems, play an important role in this scenario due to its high thermal storage capacity and small volume variation. Phase change materials (PCM) are substances which absorb or release a significant amount of energy during its phase transition, in a process at nearly constant temperature. Even though the large amount of PCM types available, ice is a favored choice in view of its low-cost, high-energy density and its melting temperature (Dincer, I., Rosen, M. 2011).

Ice storage, when integrated with heat pumps and solar collectors, represents a significant advancement in efficient energy management and sustainability within the realm of heating, ventilation, and air conditioning (HVAC) systems. This innovative combination offers several key advantages over conventional heating and cooling systems. For instance, in places where boreholes cannot be drilled, the ice storages can be used instead of ground source systems. Also, the ice storage systems can be a replacement of air source-based concepts, in cases when noise problems or efficiency are to be considered (Carbonell et al., 2015). A few examples of ice thermal storage systems being used in different types of industries, including air conditioning, food processing and building energy conservation can be found in (Zhao et al. 2020), (Sidik et al. 2018), (Bayrak et al. 2017) and (Sheikholeslami et al. 2021).

Mathematical modeling plays a crucial role in understanding and optimizing ice storage systems, offering insights into their thermodynamic behavior, performance characteristics, and operational efficiency. These models typically incorporate equations derived from principles of heat transfer, fluid dynamics, and thermodynamics to simulate the complex interactions within the system. One key aspect of mathematical modeling involves predicting the thermal behavior of ice storage mediums, such as water or phase change materials, during charging and discharging cycles (Buchner, Sebastian 2005). This includes analyzing heat transfer mechanisms, phase change phenomena, and energy storage capacity under varying operating conditions.

In the scope of this study, a mathematical model for an ice storage system will be presented and later, further integrated in an overall system containing, a single-family house, a heat pump and a solar thermal collector field. The simulations will be performed using MATLAB/Simulink (The MathWorks, Inc. 2024). The mathematical model

used in the simulations presents some simplifications to enable a faster simulation time, without losing the accuracy. In the further sections the overall system as well as the mathematical model for the ice storage will be presented and detailed. Also, the results achieved using the ice storage in the overall system, as the State of Charge (SOC) and the temperature of the ice storage will be calculated and discussed. Finally, the conclusions and the outlook for the project will be presented.

2. Methods and methodology

2.1 System overview

Figure 1 presents an overview of the entire system used in this work. It consists of an ice storage device, a buffer storage tank, a solar collector (i.a. PVT collector) and a heat pump. The evaporator of the heat pump is connected to the ice storage tank via a hydraulic circuit. The energy required for vaporization is therefore provided by the ice storage. The heat generated from the heat pump is transferred to a buffer storage, from which energy is supplied for domestic heating and domestic hot water (DHW) needed in a single-family house. Regeneration of the ice storage is done via the solar circuit. However, if there is sufficient solar radiation, the buffer storage can also be charged directly via the solar circuit. In summer, the ice storage can also be used for cooling applications. For this purpose, the energy in form of ice is used directly from the ice storage, without the use of a conventional cooling unit. Therefore, the ice storage is connected to the building's cooling distribution system via another hydraulic circuit, including an additional heat exchanger. In this work the focus is on the mathematical modelling of the ice storage system (highlighted in the blue dotted square), which will be further described in the next section.

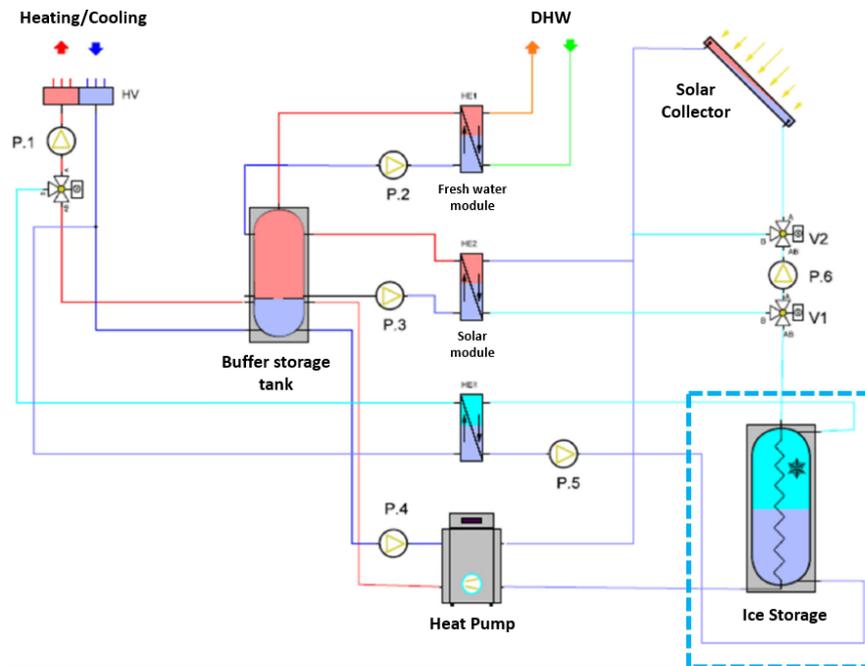


Figure 1: Overview diagram of heat pump system with ice storage and solar regeneration

2.2 Mathematical model of ice storage system

In order to develop a mathematical model for an ice storage system, it is important to understand the dynamics of it. The relationship between the media temperatures and the speed of ice formation affects the cost-effectiveness of the ice storage. Hence, although the ice formation will happen in a faster pace at low temperatures, low coefficients of performance (COP) are to be expected for the heat pumps in this condition. Another important factor is that the ice formation is a transient process, representing a heat conduction problem in which a phase change of the fluid occurs. This problem is usually referred to in the literature as Stefan-Problem, which is characterized by the fact that the heat flows occur as a result of transient heat conduction and also are directly linked to the phase change enthalpies (Dohmann 2016).

The formation of an ice layer in the system considered in this paper, happens in a thin-walled layer of stainless steel, which separates two media from each other, in this case, water and glycol. A cold flow of glycol (cooling medium) is separated from the wall of water, which is already at a solidification temperature. The growth of the ice layer is one-dimensional and occurs along the spatial coordinate x , as it can be observed in Figure 2. The layer of ice

represents a transport resistance for the heat flow between the water and the glycol.

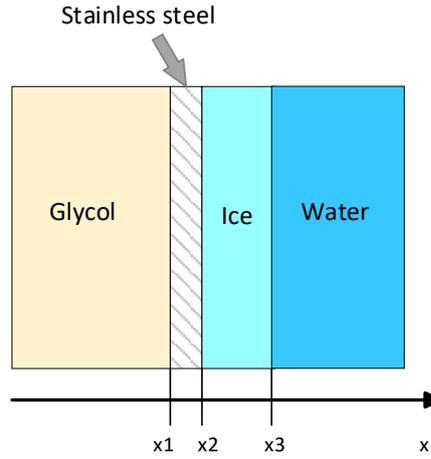


Figure 2: Example of ice formation in a heat exchanger of an ice storage system

It is also important to observe that the temperature of the ice storage depends on the heat flows to the wall and the heat flows due to the heat exchangers. For the first phase of the project, the mathematical modelling using a simpler model will be implemented in MATLAB/Simulink. In this model, the formation of ice on the heat exchangers will be calculated through the enthalpy H . This is a lumped capacitance model, meaning that the temperature inside the ice storage is assumed to be homogeneous (Winteler C. et al. 2014).

$$H_{ice} = \int \frac{1}{m_{ice}} \left(\sum_i (\dot{Q}_{2ice})_i + UA_{tank}(T_{wall} - T_{ice}) \right) \quad (1)$$

Where, m_{ice} (kg) represents the mass of liquid in the ice storage, $(\dot{Q}_{2ice})_i$ (W/m²) describes the heat flows through heat exchangers, A_{tank} (m²) is the area of the storage tank and T_{wall} and T_{ice} (°C) are the temperatures of the storage wall and the ice, respectively.

The ice storage temperature T_{ice} is calculated by means of a linear interpolation from the values in Table 1, which depends on the enthalpy (H). In the table, cp_{ice} and cp_w (J/kg K) represent the specific heat capacity of the ice and water, respectively, while q_{fice} is the specific heat of freezing water-ice.

$$T_{ice} = T_{tabelle}(H_{ice}) \quad (2)$$

Table 1: Interpolation values for temperature calculation

Temperature [°C]	Enthalpy [J/kg]
-10	$-10 * cp_{ice}$
-3	$-3 * cp_{ice}$
0	q_{fice}
10	$10 * cp_w + q_{fice}$

Using the enthalpy, the thermal energy stored during the melting phase transition and the thermal energy released in the solidification phase transition, can be taken into consideration.

In order to solve the Equation (1), the initial enthalpy value has to be calculated according to Equation (3):

$$H_{init} = (1 - f_{ice}) * 3.35 * 10^{-5} + \Theta(-f_{ice}) * 4182 * T_{init} \quad (3)$$

Where, f_{ice} (-) represents the initial fraction of ice inside the storage tank and T_{init} (°C) is the initial storage temperature, both values are user-defined initial parameters. Here, the $\Theta(z)$ is the Heaviside function, a step function which has the value of zero for a negative z and assumes a value of one for $z \geq 0$.

Further steps

In a subsequent second phase of this project, a more detailed mathematical approach will be used to describe the ice storage system. The mathematical model for the ice storage will be derived from the solution of the energy conservation law applied to the water of the storage integrated over several control volumes, as can be observed in Equation (4) (Carbonell et al. 2014), (Carbonell et al, 2015). In order to simplify the calculations, the following assumptions will be considered:

- The forced convection heat transfer between control volumes is neglected.
- Physical properties are constant in the control volumes.
- The solid phase will remain always at the same temperature.
- The viscous dissipation, radial fluid flow, compressibility, external forces and axial heat conduction are neglected.

$$\rho c_p \frac{\partial T_w}{\partial t} + (\rho c_p)_{ice} \frac{\partial T_{ice}}{\partial t} = \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{h_f}{V} \frac{\partial m_{ice}}{\partial t} + \dot{q}_{ext} + \dot{q}_{hx} \quad (4)$$

In Equation (4), t is time (s), y is the coordinate along the height of the storage, T_w (°C) is water temperature, V (m³) is the water volume of the storage, λ (W/mk) and ρ (kg/m³) are the heat conductivity and density of water respectively, c_p (J/kg K) is specific heat capacity of water, h_f (J/kg) is enthalpy of fusion, m_{ice} (kg) is mass of ice and \dot{q}_{ext} and \dot{q}_{hx} (W/m³) are the heat fluxes per unit volume between the storage fluid and the surroundings and heat exchanger respectively.

The first and second terms of Equation (4) are the accumulated sensible heat of the fluid and solid ice. On the right-hand side, the first term represents the heat of conduction between control volumes, the second is the latent heat of solidification and melting, and the last terms are the heat from the surroundings and heat exchangers respectively.

The heat loss to the surroundings through the external surface area of the tank (A_{ext} in m²), is calculated as follows:

$$\dot{Q}_{ext} = U_{ext} A_{ext} (T_w - T_{ext}) \quad (5)$$

In Equation (5), T_{ext} (°C) is the temperature of the surroundings, U_{ext} (W/m²K) is a heat transfer coefficient and $\dot{Q}_{ext} = \dot{q}_{ext} V$.

Regarding the heat transfer from heat exchanger to storage fluid, it can be calculated using the following equation,

$$\dot{Q}_{hx} = -\dot{m} c_p (T_{f,o} - T_{f,i}) \quad (6)$$

Where, \dot{m} represents the mass flow rate (kg/s) of cooling fluid, $T_{f,o}$ and $T_{f,i}$ (°C) are the fluid temperature of the heat exchanger at the outlet and inlet, respectively. In order to get to Equation (6), a constant heat transfer coefficient through the fluid path, has to be considered.

3. Results and discussion

3.1. Ice Storage description

The simulation model for the ice storage is performed to represent the ECOTHERM Ice Memory System. In this system, 12 spiral flat coil cooling bundles are connected in parallel. Figure 3 presents the ice storage system and the internal view of one of the spirals (marked in red) is amplified in the left-hand side. The storage tank is made of stainless steel, with a height of 2,9 meters, a volume of 5000 liters, and a weight of ca. 4800 kg. The total storage capacity can reach a maximum of 425 kWh at approximately 65% of icing. During the simulations, a system with 10 m³ was used, combining in parallel two of the former mentioned Ice Memory System. The material properties of water and glycol were determined using the CoolProp add-in tool in MATLAB (Bell et al., 2014).

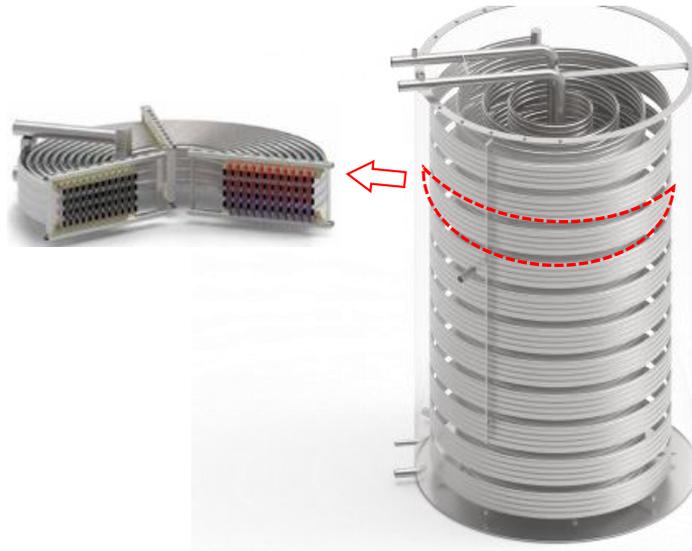


Figure 3: Ice storage system (right) and internal view of one of the cooling bundles (left)

3.2. Complete system

As previously mentioned, the ice storage model developed in this project will be integrated in a comprehensive system. Figure 4 presents the overview of the system created in Simulink, where the ice storage is the blue block outlined in red. The simulation is carried out using the Carnot Toolbox (CARNOT Toolbox, 2024), which is used to analyze the performance of the ice storage. In general, ice storages can be used for both heating and cooling. Nevertheless, in the course of this work, the performance of the ice storage only for heating purposes is analyzed. For this reason, a complete heating system is simulated in Simulink. The system consists of a single-family house, an ice storage, a heat pump and a solar thermal collector field. The heat demand of the single-family house is met by the heat pump. The ice storage is used to provide the thermal energy required for the heat pump process and consequently, the water in the ice storage decreases its temperature and freezes due to the associated heat flux. Thus, the ice storage must be regenerated. The energy required for regeneration is supplied by the solar thermal collector field. The regeneration process is controlled via the temperature difference between the collector outlet and the temperature of the medium in the ice storage. If the temperature difference is greater than 5 K, the solar pump is switched on and the ice storage is regenerated, whereas the solar pump is switched off at a temperature difference less than 1K.

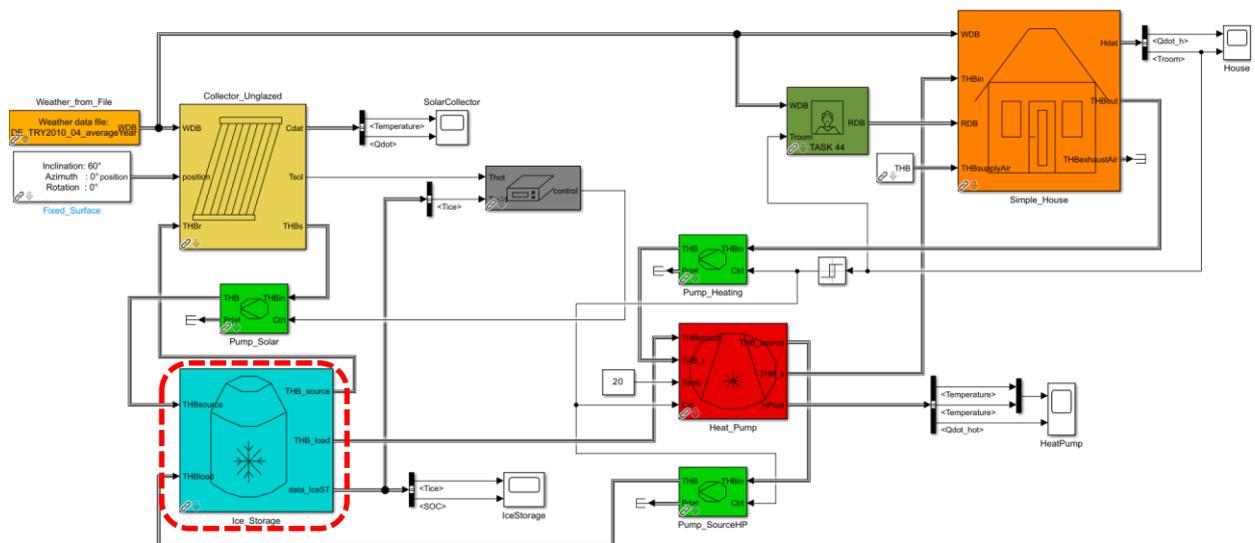


Figure 4: Simulink view of the complete system

The system parameters used in the simulation are listed in Table 2:

Table 2: Simulation parameters

Parameter	Value
Single family house	Specific heat demand 45 kWh/(m ² a), 120 m ² heated living surface
Ice storage	10 m ³
Thermal insulation surface ice storage	50 mm, 0.05 W/(m K)
Heat pump	7 kW nominal heat output
Solar thermal collector field	20 m ² unglazed

The ice storage model in the Carnot Toolbox is designed for use with underground ice storage systems. However, the ice storage under consideration in this work is intended for installation within a building. In consequence, the corresponding ice storage model has been modified so that the ice storage is now surrounded by air, as opposed to soil. The calculation of heat losses or gains to the surrounding air is computed according to Equation (5). In order to prevent condensation occurring on the exterior surface of the ice storage walls, a 50 mm layer of thermal insulation is applied to the model.

Subsequently, an annual simulation of the entire system is conducted in accordance with the previously mentioned system parameters (Table 2). Figure 5 shows the State of Charge (SOC) of the ice storage. The SOC describes how much thermal energy is stored in the ice storage, related to the energy content of the phase change. For example, a SOC of 1 indicates that the entirety of the water within the ice store is in a liquid state at 0°C. By contrast, a SOC of 0 means that all the water in the storage is ice at 0°C. Since the water can also reach temperatures above 0°C, for example during regeneration, it is possible that the SOC of the ice storage can reach values above 1. According to Figure 5, this is particularly the case in summer, as the heat pump is not in operation at this time. Due to the expansion of the ice and the associated blasting effect during the icing process, the ice storage and the thermal collector field have been designed in such a way that the degree of icing does not exceed 80%. Therefore, the minimum possible SOC is 20%. The simulation indicates that this threshold will only be reached by the end of January. Figure 5 clearly shows that, with the specified configuration of the components, the SOC reaches a value between 0.2 and 1 during the heating period. This indicates that the energy associated with the phase change of the water is optimally utilized during this period. Following the heating period, the temperature within the ice storage may reach a maximum of 30°C (Figure 6). Since the heat pump is not operating during this period, only the heat losses to the surrounding air need to be covered by the solar thermal collector. For this reason, the temperature of the ice storage varies between 20°C and 30°C during the summer. At the beginning of the heating period in autumn, the temperature in the ice storage drops accordingly as the heat pump starts up and initially ranges between 10°C and 0°C. Towards the end of the year, the water begins to freeze again. The SOC now reaches constant values below 1.

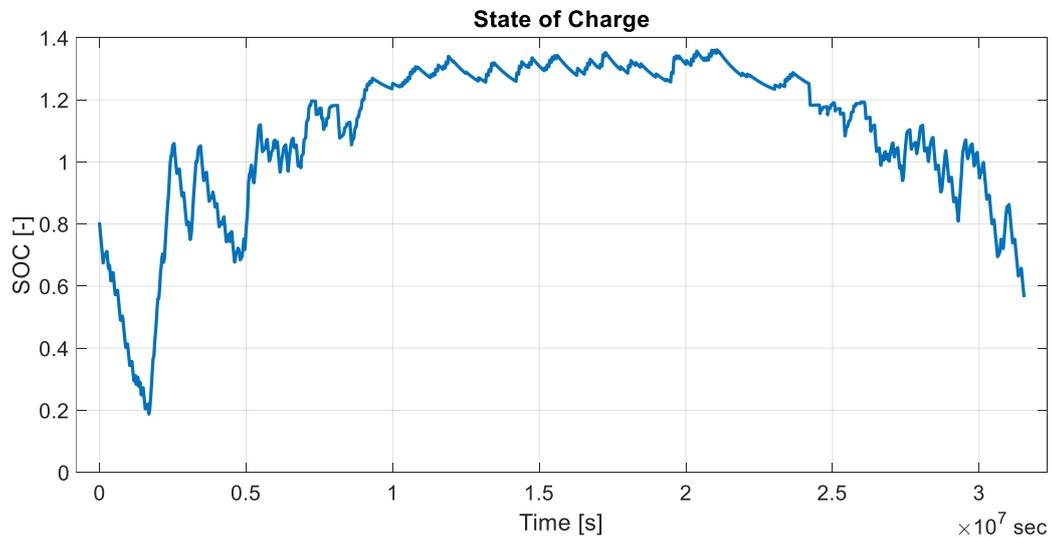


Figure 5: State of charge of ice storage

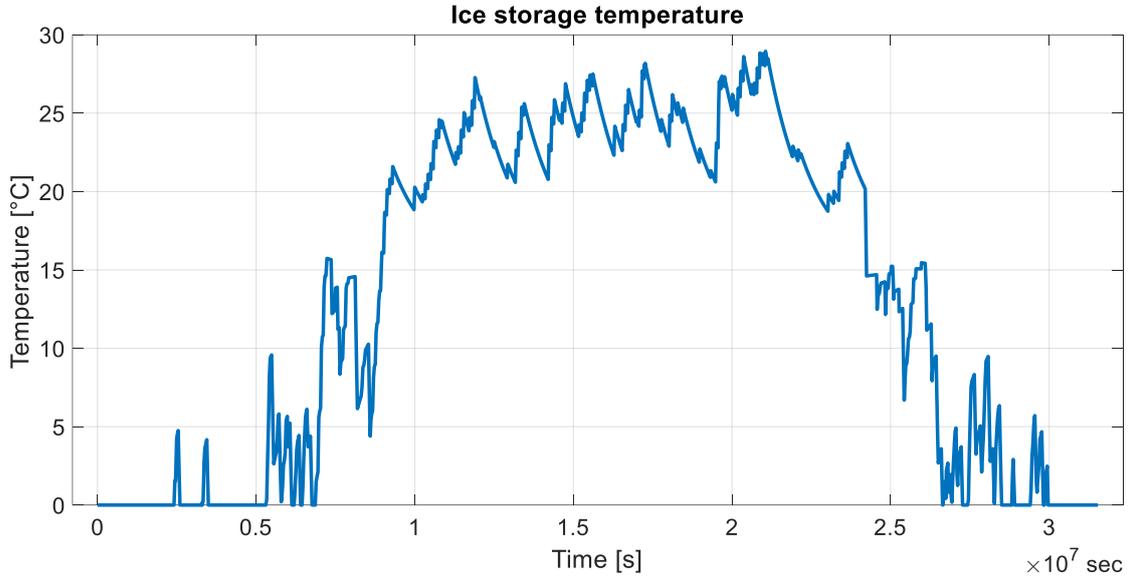


Figure 6: Temperature of ice storage

The energy input from the ice storage to the heat pump is shown in Figure 7. In total, the ice storage provides almost 6000 kWh of thermal energy to the heat pump. In the first half of the year, approximately 3300 kWh and in the second half of the year, 2700 kWh are delivered to the heat pump. The cumulated energy curve can be employed to ascertain the mean thermal power delivered by the ice storage to the heat pump. During the first part of the heating period, this equates to approximately 1.36 kW, while during the second part of the heating period, an average of 1.15 kW is observed.

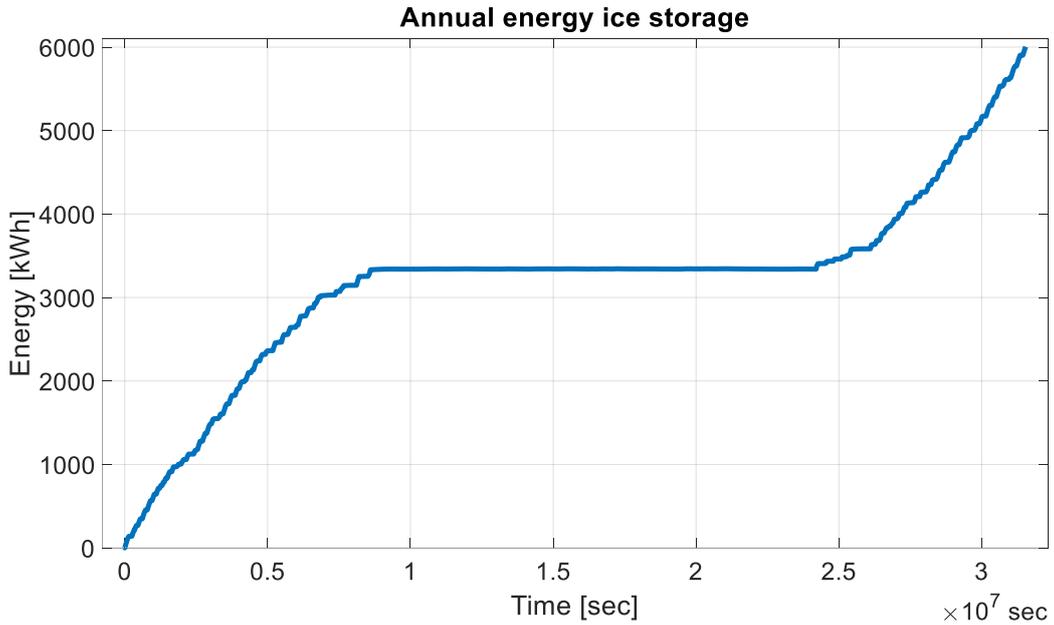


Figure 7: Cumulated energy curve of the ice storage

Several key figures are used to describe the efficiency and performance of the whole system and the individual components. The efficiency of the heat pump is described by the Seasonal Coefficient of Performance (SCOP):

$$SCOP_{HP} = \frac{Q_{H,HP}}{W_{el,HP}} \quad (8)$$

Where the $SCOP_{HP}$ is the seasonal coefficient of performance (-), $Q_{H,HP}$ (kWh) is the annual thermal energy which is supplied by the heat pump and finally $W_{el,HP}$ (kWh) is the annual electrical energy which is consumed by the heat pump.

In accordance with the parameters established for this simulation, the $SCOP_{HP}$ attains a value of 4.54. However, this value only reflects the efficiency of the heat pump and does not include other electrical consumers such as various

circulation pumps. For this reason, the performance of the overall system is described with the Seasonal Performance Factor (SPF). The SPF is defined as follows (Malenkovic et al., 2013):

$$SPF = \frac{\int (\dot{Q}_H + \dot{Q}_{DHW} + \dot{Q}_C) dt}{\int \sum P_{el} dt} \quad (9)$$

Where the $SPF(-)$ is the seasonal performance factor, \dot{Q}_H (kW), \dot{Q}_{DHW} (kW) \dot{Q}_C (kW) are the thermal powers for heating, domestic hot water and cooling, respectively, and finally $\sum P_{el}$ (kW) describes the overall power consumption of all components.

In this work, only the heating case is considered, consequently the corresponding energy for domestic hot water and cooling are zero. The overall system's SPF is 4.34, which is slightly below the $SCOP_{HP}$. This is due to the fact that all electrical consumers are taken into account in the SPF. To illustrate, the pumps utilized for regeneration of the ice storage or the circulation pump between the ice storage and the heat pump are incorporated into the calculations. Moreover, the previously mentioned energy quantities refer to the demand of the building and not to the energy quantities supplied by the components. The SPF therefore also includes the heat losses that occur in the system. The results achieved in these simulations were compared to the literature (Malenkovic et al., 2013) and it was possible to notice that they present a good fit.

4. Conclusions and outlook

This work presented a mathematical model for an ice storage system with spiral type heat exchangers. The model was performed to represent the ECOTHERM Ice Memory System. In this system, 12 spiral flat coil cooling bundles are connected in parallel. After the development of an ice storage model, it was then included in a complete heating system containing a simple house with a heating demand of 45 kWh/(m²a) and 120 m² heated living surface, a solar collector field with 20 m² of unglazed collectors and a heat pump with 7 kW of nominal heat output. As previously mentioned, the work done so far corresponds to the first phase of the project. Based on the results obtained during the simulations it was possible to observe that the mathematical model implemented, although simple in comparison with the model to be used in the next phase, presents reliable results when compared with literature data. As formerly explained, due to the expansion of the ice and the associated blasting effect during the icing process, the ice storage and the thermal collector field were designed to allow a maximum degree of icing of 80%. Therefore, the minimum possible SOC is 20%, which could also be observed in the results. The state of charge is an important parameter to consider when calculating the efficiency of the ice storage.

In order to assess the performance of the system, the Seasonal coefficient of performance of the heat pump and the Seasonal Performance Factor of the whole system were calculated. The SCOP of the heat pump presented a final value of 4.54, while the SPF had a value of 4.34. The difference between the values can be explained by the fact that in the SPF all the electrical consumers and the losses in the system are taken into consideration.

On the second phase of the project a more detailed mathematical model will be implemented using MATLAB/Simulink. The model will be based on the work of (Carbonell et al. 2014) and (Brandstätter 2023), which divide the system into several control volumes to perform the calculations. A few important assumptions in this model are that the physical properties are constant in the control volumes and that the forced convection heat transfer between control volumes is neglected. These assumptions simplify the calculations while maintaining the accuracy of the model leading to faster simulation times. This subsequent mathematical model will also be able to predict the amount of ice formed in the coil heat exchangers inside the ice storage system at a given time (icing degree), which is an important parameter to determine the efficiency of the ice storage and the complete system. Moreover, regarding the complete system, the ice storage will be used to provide not only heating during winter, but also cooling during summer to the single-family house.

In conclusion, as a last step in this project, experiments with a prototype made of stainless steel will be performed using different materials for the shell and heat exchangers, as well as different types of cooling media substances, as for example, the triethylenglycol. Based on these experiments results and the results of similar existent models in the literature, the mathematical model developed will be further on validated.

5. Acknowledgments

This work has been supported by the Government of Upper Austria in the project ‘COMPESTO – comprehensive energy storage’, Research Grant Wi-2022-600132/7-Au and in the project ‘ICE4H&C – ICE Heating and Cooling for SFH and MFH’ (FET-Future energy technologies call).

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