

# EFFECT OF ZINC OXIDE-WATER NANOFLUIDS ON THE THERMAL EFFICIENCY OF A FLAT PLATE SOLAR COLLECTOR – A COMPARATIVE ANALYSIS OF CENTRAL COMPOSITE DESIGN AND BOX BEHNKEN DESIGN

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## Abstract

Comparative analysis of response surface methodology (RSM) using central composite design (CCD) and box-Behnken design (BBD) in thermal efficiencies of a solar water heating system is undertaken. CCD and BBD were applied to data obtained from an experimental study that studied the effect of zinc oxide-water nanofluids on a pumped solar thermal system with a flat plate collector. The two methods were then compared in terms of the coefficient of determination and the P-values of the models. The results showed that Box Behnken Design yielded a model with a higher coefficient of determination ( $R^2$ ) of 85.88%, while Central Composite Design gave a coefficient of determination of 77.78%. It is concluded that the Box Behnken Design achieves a better-fitting model with fewer experimental runs. On the other hand, the CCD produced a more accurate model with a P-value of 0.023 (lower than the standard F-value of 0.05 for a confidence level of 95%) as against a P-value of 0.097 for BBD (higher than the standard F-value of 0.05 for a confidence level of 95%). Consequently, CCD is a better option when producing an accurate model while BBD is preferred for making models with a good fit.

*Keywords: Central Composite Design, Box Behnken Design, Solar Thermal, Flat Plate, Efficiency*

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## 1. Introduction

Solar thermal systems have increasingly become popular in Southern Africa because of the copious amounts of solar irradiance received in the area. These systems have proven to be effective at heating water to ideal temperatures for residential and industrial use (Joubert, et al., 2016). However, there is still hesitancy in using solar thermal systems because most of them use backup heating elements to compensate for days with little solar irradiance (Gautum & Saini, 2020). The result is that consumers still end up using electricity to heat their water on cloudy days and in winter. In a bid to curb this, numerous studies have been carried out to maximize the thermal efficiency of these systems. Some of these studies utilize nanofluids as the working fluid of the system, mainly due to their superior thermophysical properties in comparison to other working fluids such as water and water-glycol mixtures (Shojaeizadeh, et al., 2015). Studies have shown that the use of nanofluids increases the efficiency of a flat-plate solar collector (Amin, et al., 2015). A specific study done by Sindhuja et al. (2018) showed that nanoparticles can also be used in high-temperature applications such as concentrated solar power absorbers. Furthermore, some researchers have employed optimization techniques to optimize the working parameters of these systems and an example of such is Response Surface Methodology (RSM).

Response Surface Methodology is a popular optimization technique used by engineers and scientists to achieve optimal process conditions without having to carry out numerous experiments. The advantages of this method include that it is precise, saves time, and produces models with satisfactory accuracy (Esfe, et al., 2016). It applies several mathematical and statistical relationships to design experiments, regress data, and find the target response specified by the researcher. According to Chollom et al. (2020), this method produces models that are either linear or quadratic depending on the established relationship between the dependent and independent variables of the experiment.

Central Composite Design is a design of experiments that involves five levels of two to ten factors. The levels for CCD are  $+\alpha$ ,  $+1$ ,  $0$ ,  $-1$ , and  $-\alpha$ , where  $+1$  and  $-1$  are the cube points,  $0$  is the mid-point, and  $-\alpha$  and  $+\alpha$  are axial points. This design typically involves excessive combinations of factors and levels to model the data as accurately as possible (Rakić, et al., 2014). A study by Hatami & Jing (2017) successfully found the best wave

profile of aluminium oxide-water nanofluids in a wavy direct absorber solar collector. The researchers considered the amplitude of the wave and the number of waves as the independent variables of the experiment to optimize the Nusselt number of aluminium oxide-water nanofluid in the solar collector. With the use of CCD, authors were able to find the wave amplitude and number that yielded the highest Nusselt number. Another research by Ghasemi et al. (2021) employed CCD to simultaneously minimize pressure drop and maximize heat transfer in a mini channel heat sink. The explanatory variables were defined as the number of channels, mass flow rate, and channel diameter, with the response variables as the heat transfer rate between them and pressure drop. The research concluded that the optimal design consisted of four channels, a mass flow rate of 3 cm/s, and a channel diameter of 4mm, consequently, the desirability function of the optimization was 0.573.

Box Behnken Design (BBD) is another design of experiments that falls under Response Surface Methodology. It involves three levels, namely -1, 0, and +1 and it gives a similar mathematical model to that produced by CCD. The main difference between the two is that CCD involves more experimental runs than BBD because it considers the axial points of the dataset instead of solely the cube points (Rakić, et al., 2014). This also implies that BBD has a lower degree of freedom in comparison to CCD which makes it less rotatable. BBD is preferred for applications whereby the range of favourable operating conditions has already been determined and the researcher now seeks to pinpoint the most optimal conditions. In a study to enhance thermal efficiency by optimizing the filling ratio, tilt angle, and dispersion mass fraction of the nanoparticles in an evacuated tube solar collector using the Box Behnken Design (Sarafraz, et al., 2019), the researchers were able to model the thermosyphon system using RSM and found out that a maximum thermal efficiency of 96.2% was achieved when the tilt angle was 48°, with a filling ratio of 0.65, and a mass fraction of 0.3. When validation was carried out, the RSM model had a margin of error of 1.5%, proving the accuracy of the RSM-generated model. As RSM involves different Designs of Experiments, this paper specifically presents Central Composite Design (CCD) and Box Behnken Design (BBD) in the context of optimization of a pumped solar thermal system with flat plate collectors, suitable for hot water preparation at the household/domestic level

## 2. Methodology

Experiments were conducted on an existing pumped solar thermal system with a flat plate collector and zinc oxide-water nanofluids as the system's working fluid (Figure 1).



**Figure 1: Existing solar water heater used for experiments (Rabasoma et. al., 2021)**

The apparatus comprised a full port ball valve, a flat plate collector, two 100-litre storage tanks, a pump, piping, and pipe fittings. The pipes and storage tanks were all insulated to minimize thermal losses to the environment. The first portion of the experiment involved preparing the zinc oxide-water nanofluids at varying mass fractions ranging from 0.0% to 0.15%. Additionally, a concentration of 1.0% of sodium dodecyl sulphate (SDS) was used as a surfactant in the prepared nanofluids.

The working fluid's mass flow rate was also varied at 0.0556 kg/s, 0.1667 kg/s, and 0.2778 kg/s to investigate the effect of mass flow rate on the system's thermal efficiency, which was controlled by a full-port ball valve. The inlet temperature, outlet temperature, and solar irradiance were recorded and used to calculate the thermal efficiency.

The experiments investigated mass flow rate, mass fraction, and irradiance as independent variables affecting the thermal efficiency of the system and Table 1 shows the allocated values for each level. Central Composite Design and Box Behnken Design were used to generate tables for the experiments' design. Minitab software was used to generate the design tables for both methods and the same software was used to analyse the data obtained from the experiments.

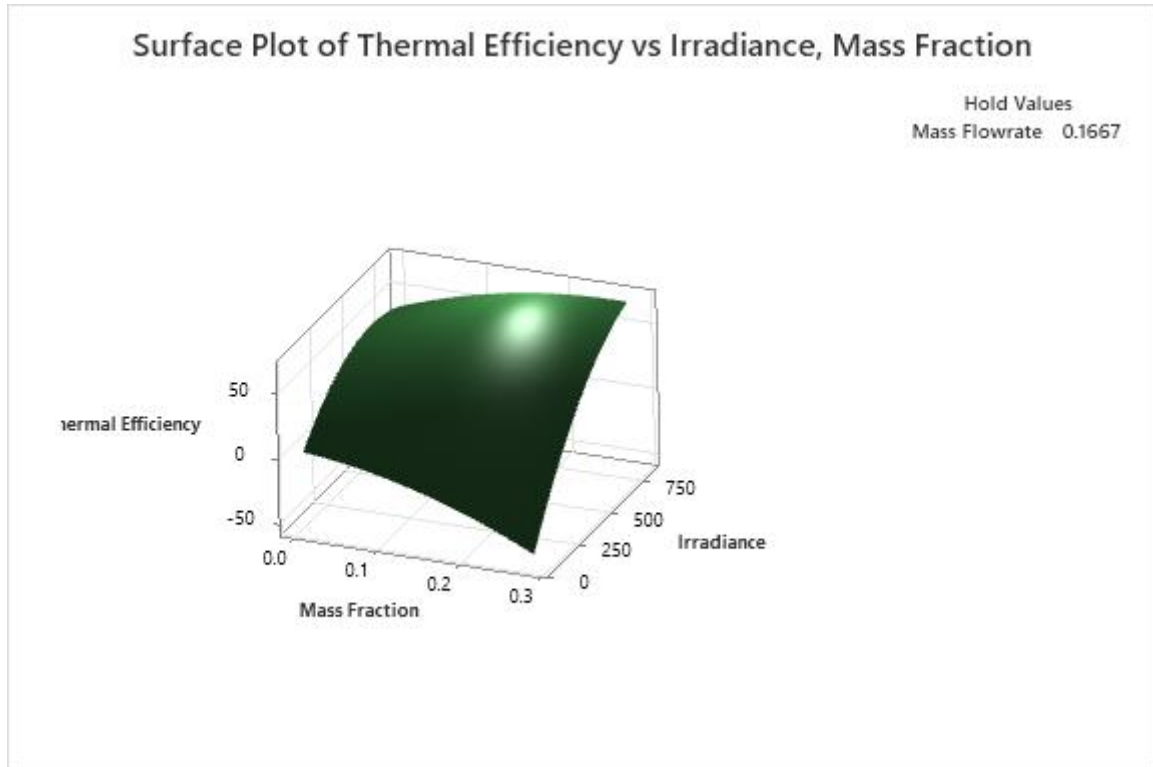
**Table 1: Specific values for the high, middle, and low levels of mass flowrate, mass fraction, and irradiance as independent variables affecting the thermal efficiency of the system**

| Factor                             | Level  |        |        |
|------------------------------------|--------|--------|--------|
|                                    | +1     | 0      | -1     |
| Mass Flowrate (kgs <sup>-1</sup> ) | 0.2778 | 0.1667 | 0.0556 |
| Mass Fraction (%)                  | 0.0    | 0.75   | 0.15   |
| Irradiance (W/m <sup>2</sup> )     | 64     | 486    | 908    |

Central Composite Design yielded 20 experimental runs, and the alpha value was specified as one for a face-centred design. The design was unblocked because there were no blocks for the data set obtained. Similarly, Box Behnken Design yielded 15 experiment runs for the three defined factors, and the design was also unblocked, similar to Central Composite Design. An Analysis of Variance (ANOVA) was then carried out on the model to determine the accuracy and correctness.

### 3. Results and Discussion

To analyse the data obtained, Minitab software was used, and Response Surface Methodology was also applied for regression of the data. The data was used to generate surface plots from the two designs of experiments (Figures 2 - 7). The obtained values show that the plots are optimized for the hold values of the three variables of mass flowrate, mass fraction, and irradiance.



**Figure 2: Surface plot of thermal efficiency generated with mass flowrate held constant at 0.1667 kg/s for CCD**

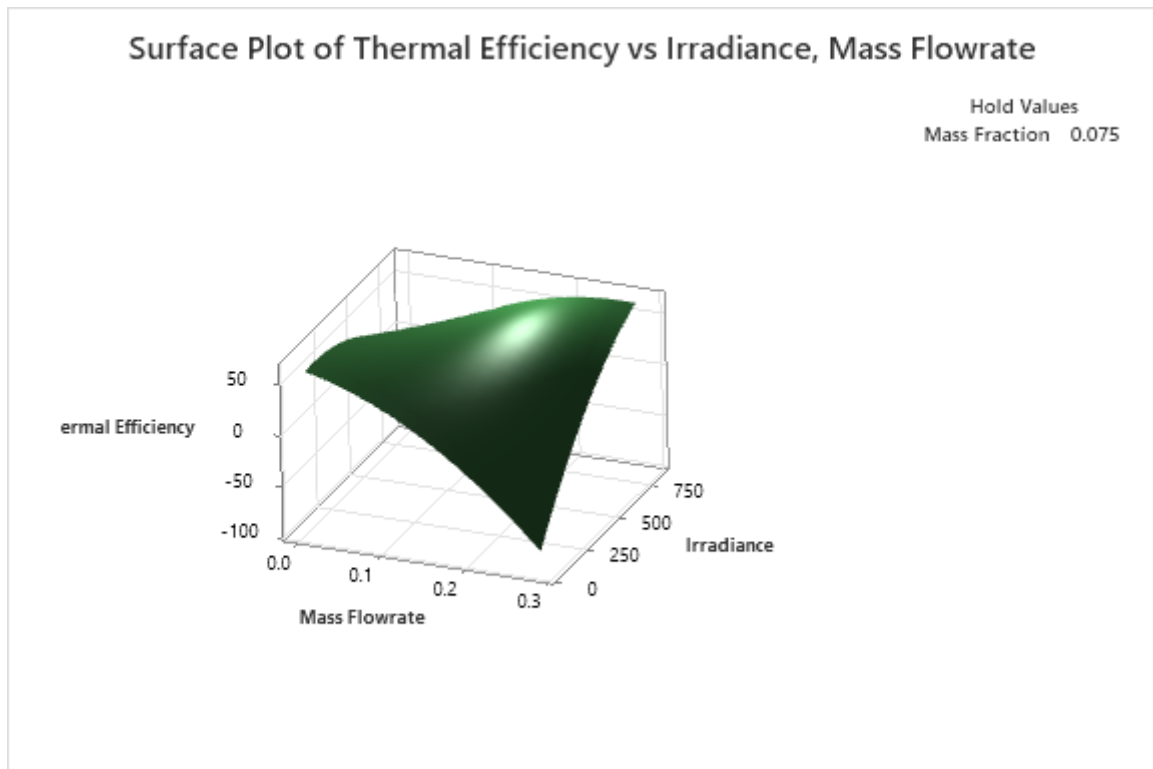


Figure 3: Surface plot of thermal efficiency generated with mass fraction held constant at 0.075% for CCD

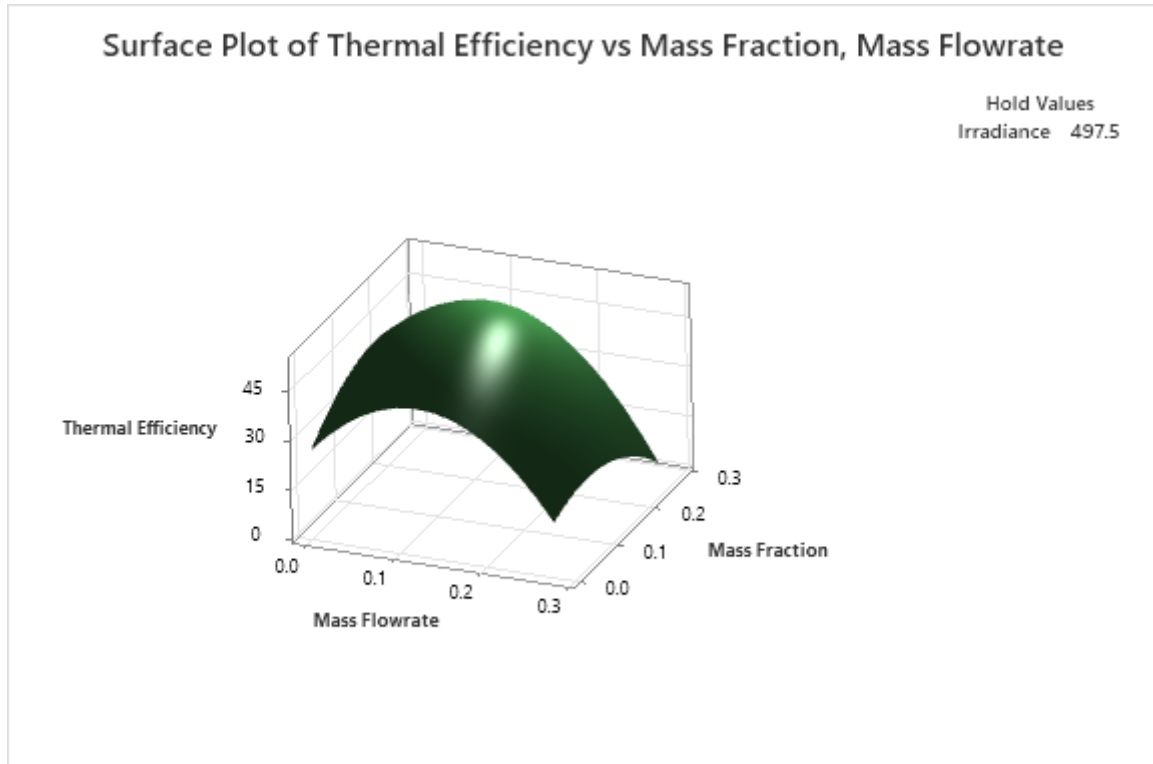


Figure 4: Surface plot of thermal efficiency generated with irradiance held constant at 497.5 W/ m<sup>2</sup> for CCD

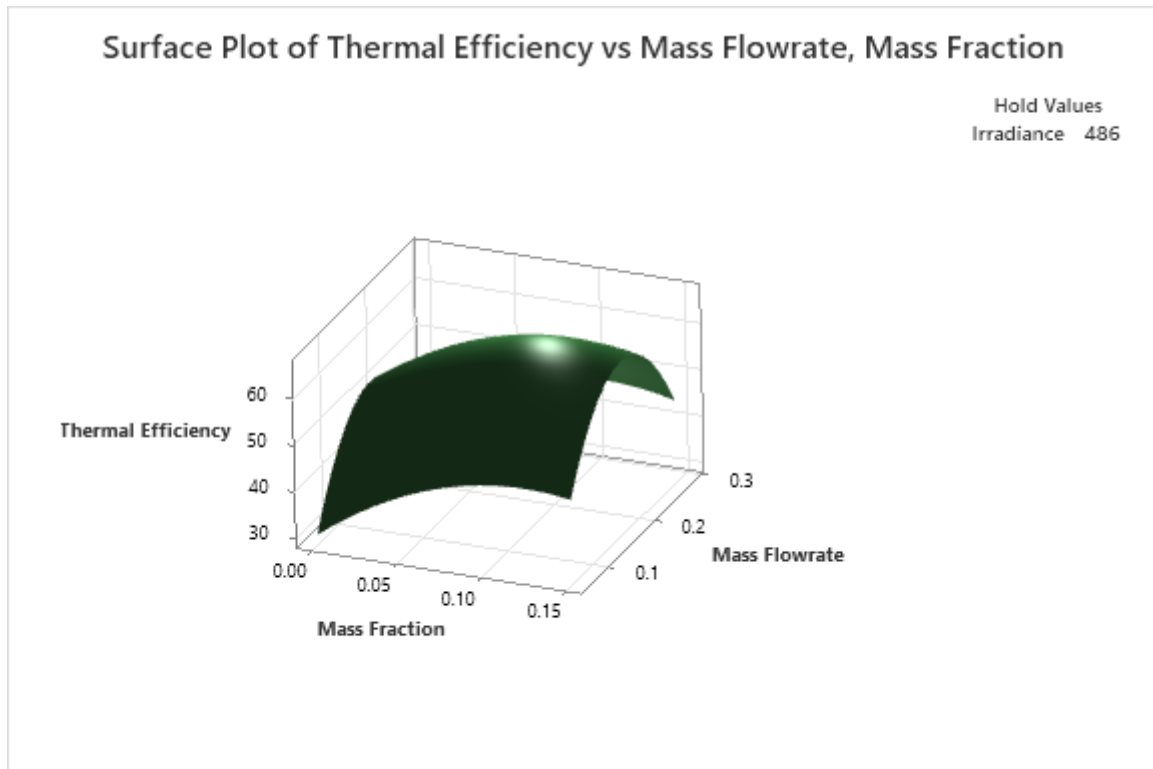


Figure 5: Surface plot of thermal efficiency generated with irradiance held constant at 486 W/ m<sup>2</sup> for BBD

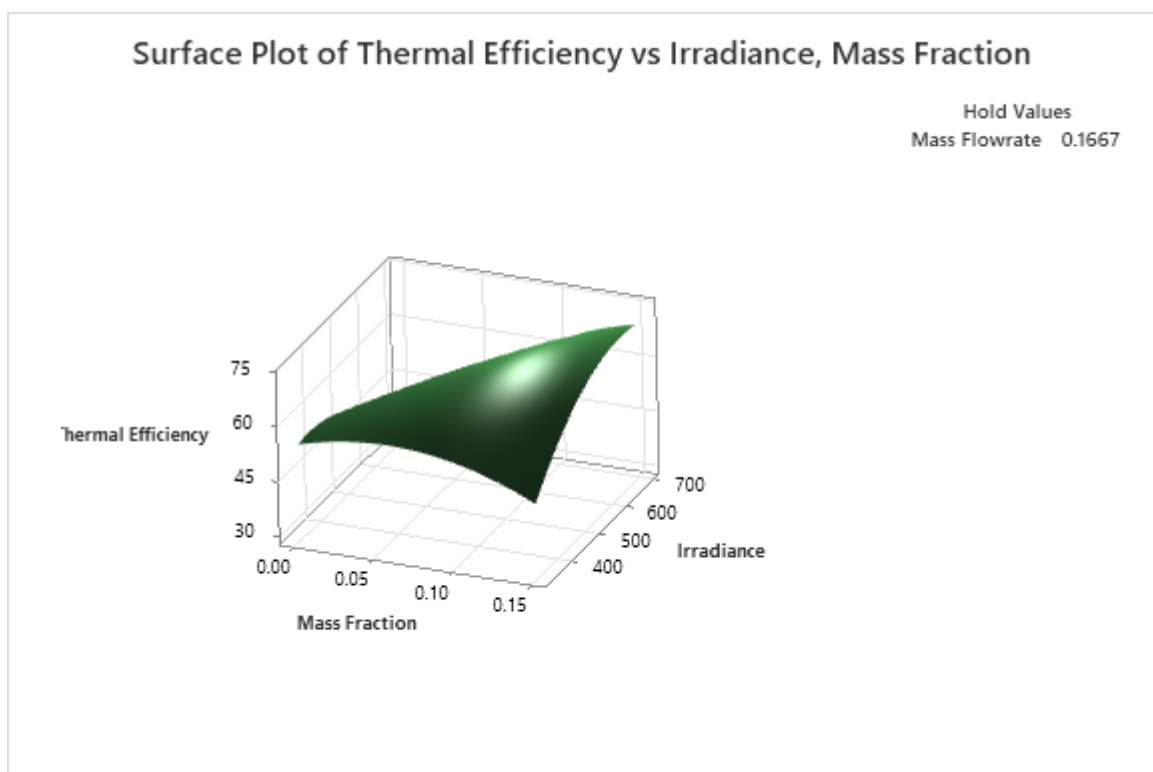
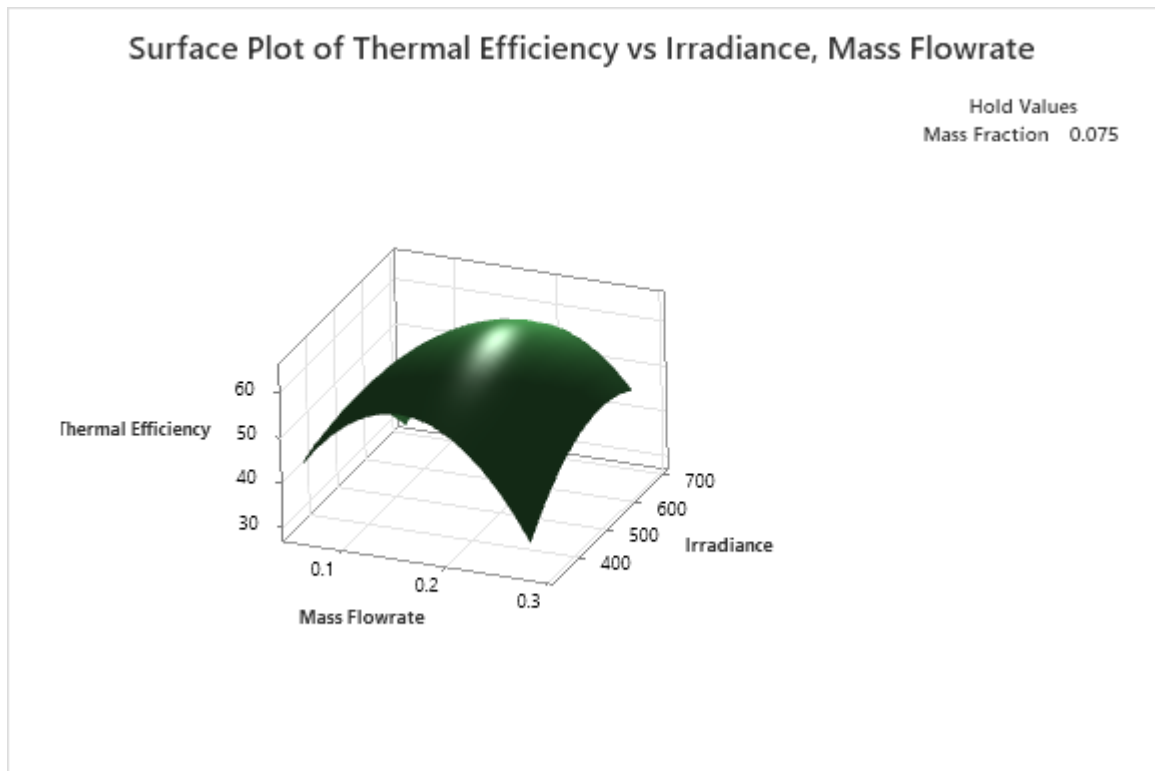


Figure 6: Surface plot of thermal efficiency generated with mass flowrate held constant at 0.1667 kg/s for BBD



**Figure 7: Surface plot of thermal efficiency generated with mass fraction held constant at 0.075% for BBD**

Table 2 presents the ANOVA for CCD while Table 3 presents the ANOVA for BBD.

**Table 2: Analysis of Variance for CCD**

| Source                       | DF | Adj SS  | Adj MS  | F-Value | P-Value |
|------------------------------|----|---------|---------|---------|---------|
| Model                        | 9  | 1956.48 | 217.387 | 3.89    | 0.023   |
| Linear                       | 3  | 800.29  | 266.763 | 4.77    | 0.026   |
| Mass Fraction                | 1  | 481.84  | 481.835 | 8.62    | 0.015   |
| Mass Flowrate                | 1  | 1.74    | 1.740   | 0.03    | 0.863   |
| Irradiance                   | 1  | 70.90   | 70.902  | 1.27    | 0.286   |
| Square                       | 3  | 459.24  | 153.078 | 2.74    | 0.099   |
| Mass Fraction *Mass Fraction | 1  | 2.68    | 2.679   | 0.05    | 0.831   |
| Mass Flowrate *Mass Flowrate | 1  | 290.54  | 290.539 | 5.20    | 0.046   |
| Irradiance*Irradiance        | 1  | 41.35   | 41.354  | 0.74    | 0.410   |
| 2-Way Interaction            | 3  | 258.14  | 86.045  | 1.54    | 0.264   |
| Mass Fraction*Mass Flowrate  | 1  | 37.11   | 37.110  | 0.66    | 0.434   |
| Mass Fraction *Irradiance    | 1  | 128.40  | 128.399 | 2.30    | 0.161   |
| Mass Flowrate *Irradiance    | 1  | 235.22  | 235.224 | 4.21    | 0.067   |
| Error                        | 10 | 558.85  | 55.885  |         |         |
| Total                        | 19 | 2515.33 |         |         |         |

**Table 3: Analysis of variance for BBD**

| Source                      | DF | Adj SS  | Adj MS  | F-Value | P-Value |
|-----------------------------|----|---------|---------|---------|---------|
| Model                       | 9  | 1895.86 | 210.651 | 3.38    | 0.097   |
| Linear                      | 3  | 284.80  | 94.935  | 1.52    | 0.317   |
| Mass Fraction               | 1  | 172.95  | 172.950 | 2.77    | 0.157   |
| Mass Flowrate               | 1  | 1.50    | 1.503   | 0.02    | 0.883   |
| Irradiance                  | 1  | 2.25    | 2.254   | 0.04    | 0.857   |
| Square                      | 3  | 475.74  | 158.581 | 2.54    | 0.170   |
| Mass Fraction*Mass Fraction | 1  | 73.30   | 73.299  | 1.18    | 0.328   |
| Mass Flowrate*Mass Flowrate | 1  | 329.29  | 329.289 | 5.28    | 0.070   |
| Irradiance*Irradiance       | 1  | 69.31   | 69.308  | 1.11    | 0.340   |
| 2-Way Interaction           | 3  | 168.88  | 56.293  | 0.90    | 0.502   |
| Mass Fraction*Mass Flowrate | 1  | 5.78    | 5.784   | 0.09    | 0.773   |
| Mass Fraction*Irradiance    | 1  | 158.98  | 158.984 | 2.55    | 0.171   |
| Mass Flowrate*Irradiance    | 1  | 39.83   | 39.829  | 0.64    | 0.460   |
| Error                       | 5  | 311.75  | 62.349  |         |         |
| Total                       | 14 | 2207.61 |         |         |         |

An analysis of the variables as presented in Table 2 shows that the Central Composite Design was able to accurately optimize the mass fraction, mass flow rate, and solar irradiance. The coefficient of determination ( $R^2$ ) was 77.78 %, while the adjusted  $R^2$  and predicted  $R^2$  values were 57.79 % and 0.0% respectively. The coefficient of determination indicated that the model produced was a good fit for the data obtained from the experiment. The P-value of the model was found to be 0.023, which is lower than the standard F-value of 0.05 for a confidence level of 95%. This shows that the model produced is significant and can therefore be used to predict the behaviour of the system under different conditions accurately.

Similarly, the analysis of variance as presented in Table 3 shows that the Box Behnken Design was also able to optimize the mass fraction, mass flow rate, and solar irradiance. The coefficient of determination ( $R^2$ ) was found to be 85.88 %, which is higher than that of the CCD model. This difference implies that the BBD model fits the provided data better than the CCD model. Additionally, the adjusted  $R^2$  and predicted  $R^2$  were 60.46% and 0.0% respectively. The coefficient of determination indicated that the model produced was also a good fit for the data obtained from the experiment. The P-value of the model was found to be 0.097, which is higher than the standard F-value of 0.05 for a confidence level of 95%. This shows that the model produced has a confidence level of at most 90% as opposed to the conventional 95%. It can be inferred that the CCD model is more significant and therefore has superior accuracy.

Table 4 presents the model summary for CCD and BBD.

**Table 4: Model summary for CCD and BBD**

| Model | S       | R-sq   | R-sq(adj) | R-sq(pred) |
|-------|---------|--------|-----------|------------|
| CCD   | 7.47560 | 77.78% | 57.79%    | 0.00%      |
| BBD   | 7.89615 | 85.88% | 60.46%    | 0.00%      |

The regression outcomes for evaluating the thermal efficiency of the system for both experimental designs – CCD and BBD are presented. Equation 1 presents the regression outcome for CCD, while Equation 2 presents the regression outcome for BBD.

$$\eta_{th} = 49.8 - 90 m_i + 45m - 0.0257H - 154m_i(m_i) - 950m(m) - 0.000077 H(H) - 317 m_i(m) + 0.485 m_i(H) + 0.542 m(H) \quad (\text{eq.1})$$

$$\eta_{th} = 3.5 - 116 m_i + 388 m + 0.111 H - 1123 m_i(m_i) - 1554m(m) - 0.000223H(H) - 146 m_i(m) + 0.824 m_i(H) + 0.305 m(H) \quad (\text{eq.2})$$

(Where  $m$  = mass flowrate;  $m_i$  = mass fraction;  $H$  = Irradiance).

#### 4. Conclusion

CCD and BBD can both be used to effectively optimize the thermal efficiency of a solar thermal system, as demonstrated in this study. The Box Behnken Design was found to surpass the Central Composite Design in terms of model fitting (86% against 78%). However, the Central Composite Design produced a more accurate model with a P-value of 0.023 (lower than the standard F-value of 0.05 for a confidence level of 95%) as against a P-value of 0.097 for BBD (higher than the standard F-value of 0.05 for a confidence level of 95%). The coefficient of determination for BBD was higher than that of CCD by 8.1%.

Both models displayed the same patterns of significant factors and interactions in the regression models. However, Box Behnken Design was observed to be a more cost-effective and accurate method because it involves fewer runs (15 runs against 20 runs for CCD) and it still produces a model that is representative of the actual system.

#### 5. Appreciation

This work was carried out under the auspices of the SOLTRAIN+ project – a regional initiative on capacity building and the demonstration of solar thermal systems in the Southern African region. For details, see [www.soltrain.org](http://www.soltrain.org)

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