Methodological Considerations for PV Power Fluctuation Characterization

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Abstract

This study critically evaluated the methodology proposed by Mazumdar et al. (2014) for characterizing power fluctuations in photovoltaic systems using a logistic distribution. Both the original parameter estimation method and a maximum likelihood estimation (MLE) approach were applied to assess the characterization method under its best possible conditions, with the evaluation supported by experimental data. The results indicate, with 95% confidence, that the logistic distribution fails to adequately model power fluctuations, particularly in the tail regions where extreme events occur. Additionally, this work highlights limitations in commonly used metrics for evaluating fit quality. Based on these findings, it is recommended that future studies employ comprehensive histogram analyses (on a semilogarithmic scale), broad quantile comparisons, and rigorous hypothesis testing, such as Pearson's χ^2 test. The evaluation guidelines proposed here can serve as a reference for future comparative studies of characterization methods in the literature.

Keywords: PV power ramps, ramp rate, power fluctuations, maximum likelihood

1. General context

Probability Density Functions (PDFs) are mathematical functions used to describe the probability density of a continuous random variable over a range of values. In a PDF, the total area under the curve across the entire domain $(-\infty, +\infty)$ equals one, meaning that the integral of the PDF over any given interval represents the probability that the random variable falls within that interval (Billingsley, 1995). The most well-known PDF is the Gaussian distribution (also known as Normal distribution), which commonly arises in naturally occurring phenomena, such as human height distributions in a population and load uncertainty in power systems (Brainard and Burnmaster, 1991; Aien et al., 2015). Beyond the Gaussian distribution, many other, less familiar but equally valuable distributions can be used to model various physical processes.

Photovoltaic (PV) power fluctuations are particularly important to model, as their stochastic nature increases the complexity of generation forecasting, system dispatch, and raises concerns related to power quality (Asensio et al., 2021). To properly characterize these power fluctuations, it is useful to analyze increments. Consider a set of power measurements at a given location, denoted by $P = \{P_1, P_2, ..., P_N\}$. The corresponding set of power increments is given by $\Delta P = \{P_2 - P_1, P_3 - P_2, ..., P_N - 1\}$, calculated over a time interval Δt. This set of increments can be plotted as a histogram, which, when normalized by probability density, results in the empirical PDF of power fluctuations at that location. For small and medium-scale generation systems with short sampling intervals, the resulting PDF often resembles a 90°-rotated curly bracket (Anvari et al., 2017; de Souza et al., 2024), as shown in Fig. 1.

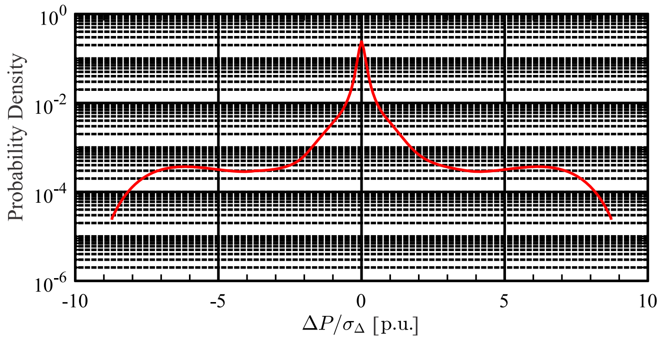

Fig. 1: The general shape of the experimental PDF for power fluctuations in a PV system, where σ_Δ represents the standard deviation of the power increments.

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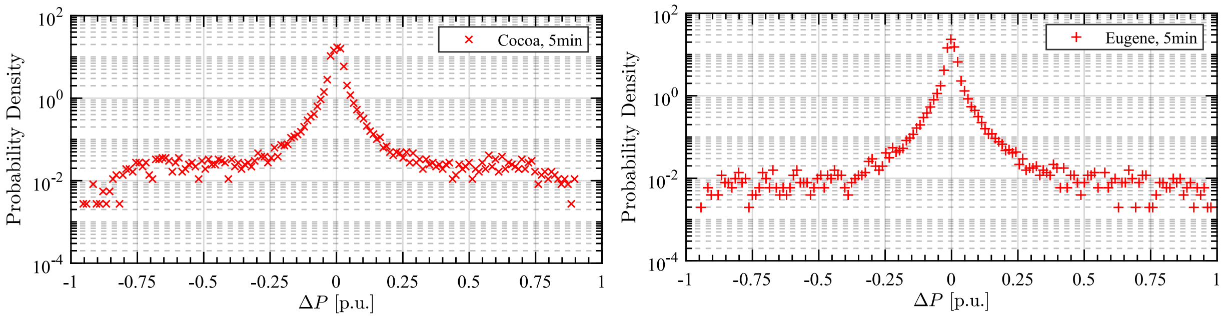
An attempt to model PV power fluctuations using probability distributions was presented by Mazumdar et. al (2014). Rather than fitting the empirical PDF directly, the authors approximated the empirical Cumulative Distribution Function (CDF), the integral of the PDF, with a Sigmoid function. Although their work represents a significant contribution to this field of research, it suffers from methodological limitations that recur in subsequent studies. Therefore, the present paper focuses on those shortcomings and introduces a more robust methodology to guide future research in the field. To achieve this objective, open-access data from the National Renewable Energy Laboratory (NREL) are used to compare the original Mazumdar et al. (2014) approach with the one proposed here.

2. Dataset details

This study uses data from two distinct installations: the Florida Solar Energy Center (FSEC) — located in the city of Cocoa – and the University of Oregon (UO) — located in the city of Eugene, both of which are part of the project 'Data for Validating Models for PV Module Performance', coordinated by NREL (Marion et al., 2014), and which have previously been used in other academic studies such as de Souza et al. (2023) and Cavalcante et al. (2024). Photovoltaic modules of different technologies were analyzed, but for this work monocrystalline silicon (mSi) modules were selected due to their widespread commercial use. Measurements were collected for at least one full year at each site, using identical modules, instrumentation, and calibration procedures to ensure data reliability. The sites were chosen to capture contrasting climatic conditions, humid subtropical (FSEC) and marine West Coast (UO), while also ensuring that the underlying data are publicly available, thus supporting climatic diversity and reproducibility of our results. Considering a sampling time of 5 minutes, the standard deviations of power fluctuations in Cocoa and Eugene are 0.103 and 0.075 p.u., respectively. The empirical probability distributions are shown in Fig. 2a and Fig. 2b. Note that, in this case, the data are not normalized by their standard deviation. Additional dataset details are presented in Tab. 1.



(a) (b)

Fig. 2: Empirical PDFs of power fluctuations in (a) Cocoa and (b) Eugene for a sampling time of 5 min.

Tab. 1: Dataset details.

Site Name	Characteristics of the datasets					
	P_{rated} (W)	Approx. location coordinates	Period of measurement	Sampling time	# of data points	Module technology
Cocoa	46.1	28.39 N, 80.75 W	Jan 21, 2011-Feb 24, 2012	5 min	35,841	mSi
Eugene	46.1	44.05 N, 123.07 W	Dec 20, 2012-Jan 19, 2014	5 min	39,357	mSi

3. Fitting data to a distribution

There are many possible ways to approximate experimental data and many more to assess the quality of such approximations. In this study, the PDF proposed by Mazumdar et al. (2014) is employed in its best possible form. To that end, in addition to using supplementary methods for validating the results, the tool used for the approximation itself was also replaced. Nevertheless, it is important not to lose sight of the main objective of this work, which is to encourage the use of more rigorous methods for evaluating the quality of fitted results, regardless of the fitting technique employed. The traditional methodology described in (Mazumdar et. al, 2014) is referred to here as the "established methodology", whereas the proposed methodology encompasses both the alternative estimation method and new metrics for validating it. In both cases, the same PDF is employed.

3.1. Established methodology

In (Mazumdar et. al, 2014), the authors propose modeling power fluctuations by fitting a sigmoid function to the experimental CDF, defined by the following expression:

$$F_{\text{sigmoid}}(x) = \frac{1}{1+\alpha e^{-\beta x}}, \qquad \text{(eq. 1)}$$

where the constants α and β control the shape of the sigmoid curve. These parameters are estimated by minimizing the fitting error with respect to the empirical CDF. Validation is performed primarily through visual inspection of the fitted versus empirical CDF and a limited quantile comparison. Other works have supplemented these checks with histogram comparisons (Keeratimahat et al., 2019), which is also included here to represent the established approach.

3.2. Proposed methodology

Although Mazumdar et al. (2014) did not state it explicitly, their fitting process using the sigmoid function is analogous to employing a logistic distribution, whose CDF is given by:

$$F_{\text{logistic}}(x) = \frac{1}{1+e^{-\frac{x-\mu}{s}}}, \qquad \text{(eq. 2)}$$

where μ (location) and s (scale) are the distribution parameters. When comparing Eq. (2) with the sigmoid in Eq. (1), it reveals the correspondence:

$$\alpha = e^{\frac{\mu}{s}}, \quad \beta = \frac{1}{s}. \qquad \text{(eq. 3, 4)}$$

Instead of relying on error minimization, the proposed methodology determines μ and s via Maximum Likelihood Estimation (MLE), providing statistically efficient and less biased parameter estimates. This fitting strategy is validated through several criteria: CDF comparison; histogram comparison; detailed quantile analysis; and hypothesis testing (Pearson's $\chi 2$ test). With the exception of hypothesis testing, the other methods are either self-explanatory or will become clear to the reader in the results section. It is therefore appropriate to explain the hypothesis testing procedure in greater detail. Pearson's χ^2 test, proposed by Pearson (1992), belongs to a class of tests that can be employed, among other applications, to perform goodness-of-fit tests. Goodness-of-fit tests allow the observer to assess, through numerical indicators, how well the frequencies of a given sample set can be modeled by a theoretical distribution.

In hypothesis testing, the analysis is based on a null hypothesis, denoted by H_0, and one or more alternative hypotheses, denoted by H_1, H_2, and so forth. The hypotheses are evaluated based on a calculated test statistic (T), a pre-established confidence level, and the problem's degrees of freedom (v). In the case of Pearson's χ^2 test, the test statistic is computed by summing the χ^2 values for each of the i events, according to Eq. (5):

$$T = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}, \qquad \text{(eq. 5)}$$

where O_i is the number of observations of type i, E_i is the expected frequency for a given sample of type i — what constitutes each sample category will become clearer in the results section —, and n is the number of categories. The degrees of freedom are defined as:

$$v = n - (1 + n_p), \qquad \text{(eq. 6)}$$

where n_p is the number of parameters being estimated. The next step of the test consists of comparing the value of T to a critical value, obtained by examining the quantiles of a χ^2. If T is greater than the critical value, H_0 must be rejected. In the context of this study, the problem can be stated as follows:

- Assumptions: (i) the data were sampled without bias; (ii) the events are mutually exclusive; (iii) the observed events are independent.
- H_0: The probability distribution of PV power fluctuations can be modeled using a logistic distribution.
- H_1: The null hypothesis is false.

In the following section, the presented methodologies will be applied to the experimental datasets from Cocoa and Eugene.

4. Results and discussion

In the established methodology, α and β parameters are estimated using the *fminsearch* function in MATLAB, with both parameters being initialized as 1. In Cocoa, the estimated values for $p = \{\alpha, \beta\}$ were {1.03, 7.29}, while in Eugene the values were {0.91, 6.54}. For the MLE estimation, the *fitdist* function, also from MATLAB, was used. However, only the scale parameter was estimated, since there is sufficient evidence in both the literature and the experimental data to infer that the distribution is symmetric around zero. The estimated values for each of the models, converted using Eq. (3) and Eq. (4), are presented in Tab. 2. It is worth noting that the data were normalized by the local standard deviation (σ_Δ) in all fittings.

Tab. 2: Parameters resulting from the fitting methods.

Site Name	Parameters	Established Sampling time	Proposed Module technology
Cocoa	μ	4.18×10^{-3}	0 (assumed)
	s	1.37×10^{-1}	2.81×10^{-1}
Eugene	μ	-1.37×10^{-2}	0 (assumed)
	s	1.53×10^{-1}	2.82×10^{-1}

Fig. 3 and Fig. 4 present a comparison between the empirical CDFs and the fitted models for the Cocoa and Eugene datasets. Considering the Cocoa data, for example, it can be observed that the model based on the established methodology adequately captures power fluctuations for small deviations, but begins to diverge significantly for deviations above $0.3\sigma_\Delta$; as the considered deviations increase, the model increasingly underestimates their probability of occurrence. A similar trend is already evident in the original study by Mazumdar et al. (2014). In the MLE-based model, there is an overestimation of the occurrence probability for fluctuations up to approximately $1\sigma_\Delta$, and an underestimation for larger fluctuations, although less pronounced than in the established methodology. An analysis of the Eugene data leads to similar observations.

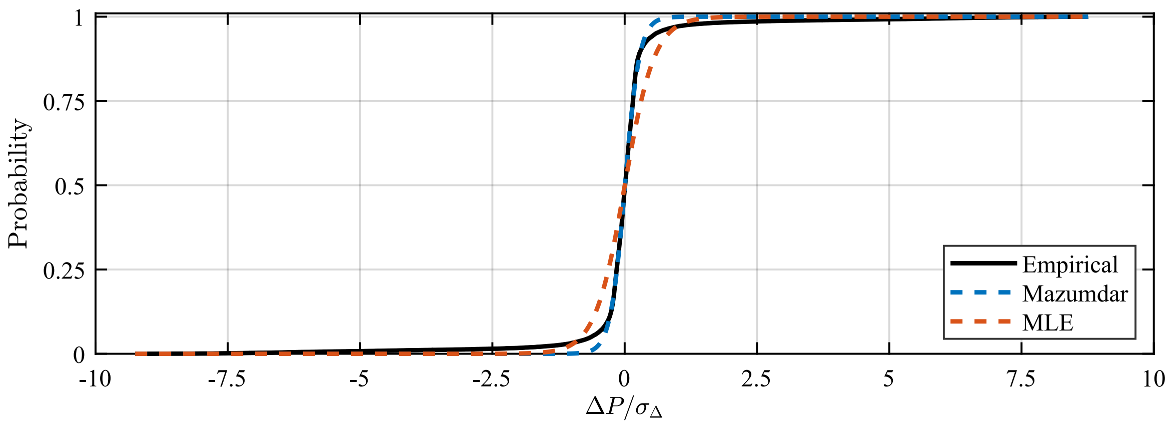


Fig. 3: Empirical and fitted CDFs of power fluctuations in Cocoa for a sampling time of 5 min.

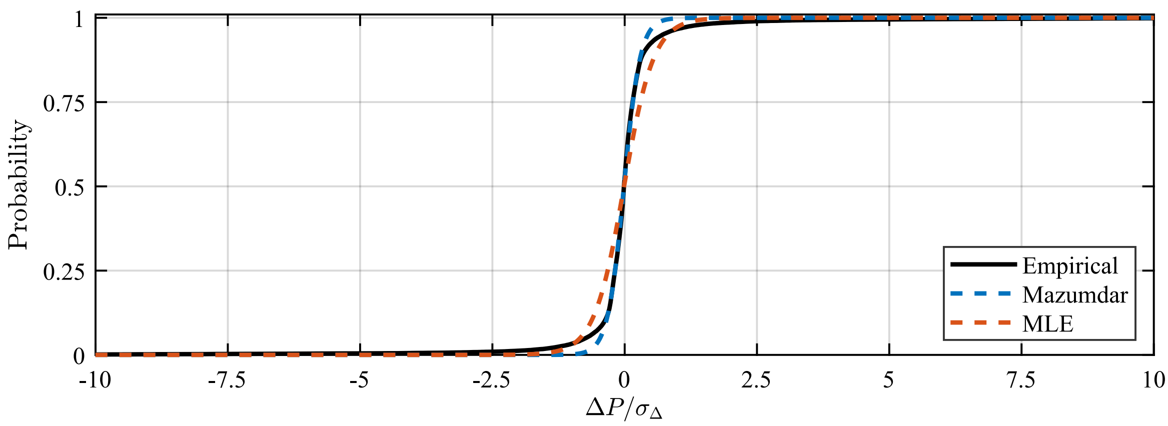
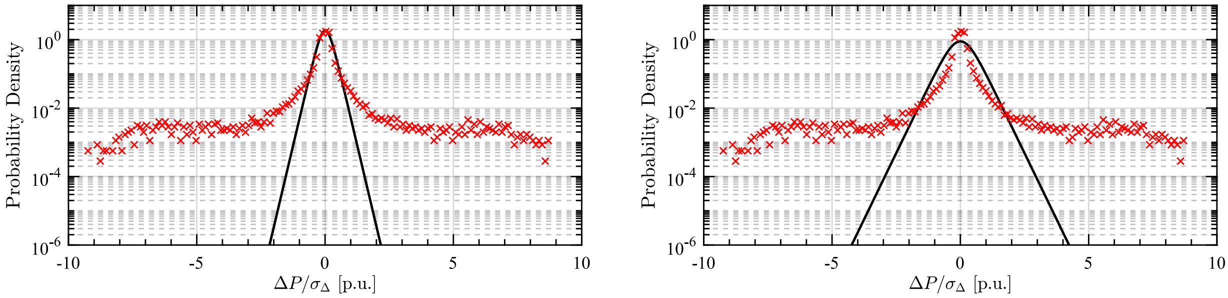


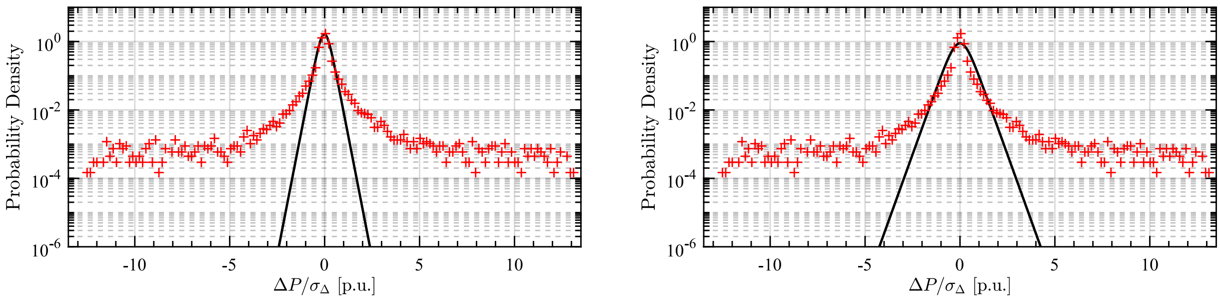
Fig. 4: Empirical and fitted CDFs of power fluctuations in Eugene for a sampling time of 5 min.

The results can also be compared using histograms, as illustrated in Fig. 5a and Fig. 5b for Cocoa and in Fig. 6a and Fig. 6b for Eugene. At first glance, it may appear evident that the models are insufficient to adequately represent the experimental distribution, but some caution is warranted. Visual analysis of histograms is highly susceptible to errors, as the chosen number of bins can significantly affect how the data appear in the figure, potentially giving the impression of a better or worse fit than is actually the case. Another point of attention involves the scale used, since a plot represented on a linear scale can give the impression of a good fit. For example, the same plot from Fig. 5a is represented on a linear scale in Fig. 7, which could give the reader a completely different impression. Examples of using histograms with a linear scale can be found in the characterization literature, such as in (Keeratimahat et al., 2019) and (Lee and Baldick, 2012).



(a) **(b)**

Fig. 5: Empirical and fitted PDFs of power fluctuations in Cocoa for a sampling time of 5 min.



(a) **(b)**

Fig. 6: Empirical and fitted PDFs of power fluctuations in Eugene for a sampling time of 5 min.

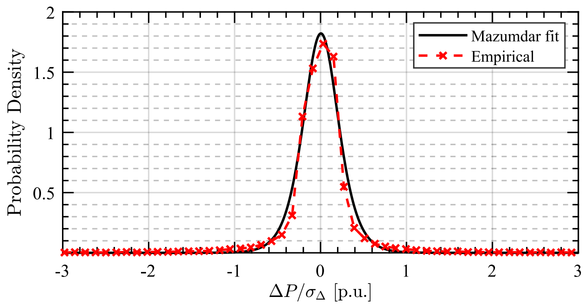


Fig. 7: Empirical and fitted PDFs of power fluctuations in Cocoa for a sampling time of 5 min – linear scale.

In (Mazumdar et al., 2014), the fitting results were also evaluated through a comparison between the quantiles of the experimental distribution and those of the proposed model. The definition of which specific quantiles to compare is arbitrary, but it is good practice to consider the distribution as a whole, or at least the intervals of interest. In the context of this work, tail events — that is, those representing large multiples of the standard deviation — are of greater interest, as they have a stronger impact on the grid. A comparison of this nature, encompassing both small and large deviations, can be seen in Tab. 3 (Cocoa) and Tab. 4 (Eugene). As previously discussed during the CDF analysis, it can be observed that the model proposed by Mazumdar et al. (2014) accurately represents only small fluctuations, which also persists in the fit performed with MLE, albeit to a lesser extent. For instance, variations of only $3\sigma_\Delta$, which may occur several times within a single day of operation, are underestimated by approximately 6000:1 in the model by Mazumdar et al. (2014) and by 15:1 in the MLE fit. In the original paper, the choice of quantiles restricted to regions of small fluctuations ultimately conveyed an impression of a better fit than was actually the case; an equivalent demonstration would be to show only the first four rows of Tab. 3 or Tab. 4, concealing the regions where the model performs poorly.

Tab. 3: Distribution of PV power fluctuations, actual and estimated probabilities (data from Cocoa).

x	$P(\|\Delta P\| > x)$ [%]		
	Empirical	**Mazumdar et al. fit**	**MLE fit**
$0.01\sigma_\Delta$	96.8	96.4	98.2
$0.05\sigma_\Delta$	81.7	82.0	91.1
$0.1\sigma_\Delta$	66.2	65.1	82.4
$0.5\sigma_\Delta$	12.3	5.1	28.8
$1\sigma_\Delta$	6.1	1.4×10^{-1}	5.5
$2\sigma_\Delta$	3.4	9.4×10^{-5}	1.6×10^{-1}
$3\sigma_\Delta$	2.5	6.4×10^{-8}	4.6×10^{-3}
$5\sigma_\Delta$	1.5	3.3×10^{-14}	3.7×10^{-6}
$7\sigma_\Delta$	5.1×10^{-1}	Approx. 0	3.0×10^{-9}

Tab. 4: Distribution of PV power fluctuations, actual and estimated probabilities (data from Eugene).

x	$P(\|\Delta P\| > x)$ [%]		
	Empirical	**Mazumdar et al. fit**	**MLE fit**
$0.01\sigma_\Delta$	96.8	96.7	98.2
$0.05\sigma_\Delta$	80.2	83.8	91.1
$0.1\sigma_\Delta$	65.5	68.5	82.4
$0.5\sigma_\Delta$	15.0	7.4	29.0
$1\sigma_\Delta$	6.6	2.9×10^{-1}	5.6
$2\sigma_\Delta$	2.6	4.2×10^{-4}	1.7×10^{-1}
$3\sigma_\Delta$	1.6	6.1×10^{-7}	4.8×10^{-3}
$5\sigma_\Delta$	9.3×10^{-1}	1.3×10^{-12}	3.9×10^{-6}
$7\sigma_\Delta$	6.1×10^{-1}	Approx. 0	3.3×10^{-9}

Up to this point, the methods employed to compare the fitted models, although data-driven and rigorously conducted, do not allow for an objective assessment. To achieve this, a goodness-of-fit test such as Pearson's $\chi 2$ test can be applied. As previously mentioned, this test relies on two hypotheses: the null hypothesis, which assumes that power fluctuations in photovoltaic (PV) systems follow a logistic distribution, and the alternative hypothesis, which rejects this assumption. To perform the test, the data samples must be grouped into categories. In the context of power fluctuations, these categories represent intervals of variation intensity. For instance, one could divide the variations into two groups — positive and negative values — and compare the frequencies predicted by the logistic distribution for each group with those actually observed in the dataset.

Nonetheless, the aim is to perform this comparison in a more informative and representative manner, which would not be achieved with only two categories. A natural consequence of this goal would be to increase the number of intervals as much as possible; however, this too would be inappropriate, since it is generally recommended that Pearson's $\chi 2$ test be applied only when the expected frequency in each category is at least 5. In this study, Scott's rule was adopted to define the initial number of categories. When it was found that some of them had expected frequencies below the threshold of 5, symmetric merging of the outermost categories was performed until all met the minimum requirement. As the model proposed by Mazumdar et al. (2014) imposes a more restrictive structure — and given that the number of bins should ideally remain consistent across all models being compared — the categories defined for this fit were extrapolated to the model obtained through MLE.

This procedure resulted in 24 categories, listed in Tab. 5 (Cocoa dataset). For example, category 1 includes deviations smaller than -1.20 and greater than -9.25. Among the 35,841 Cocoa samples, 783 fell within this interval. When evaluating the same range using the probability density functions (PDFs) of the models, the expected frequencies were found to be 5.66 and 497.76. The corresponding results for Eugene are presented in Tab. 6. Based on these values, the test statistic T is computed according to Eq. (5) and compared with the critical value (T_C) associated with the $\chi 2$ distribution at a 95% confidence level. The results obtained are summarized in Tab. 7.

Tab. 5: Observed and estimated frequencies in a dataset comprising 35,841 samples (data from Cocoa).

Bin edges	O_i	Mazumdar et al. fit		MLE fit	
		E_i	$(O_i - E_i)^2 / E_i$	E_i	$(O_i - E_i)^2 / E_i$
Bin 1: [-9.25, -1.20]	783.00	5.66	106744.07	497.76	163.46
Bin 2: [-1.20, -1.09]	74.00	6.85	658.62	230.81	106.54
Bin 3: [-1.09, -0.98]	101.00	15.12	487.70	334.62	163.11
Bin 4: [-0.98, -0.87]	122.00	33.37	235.35	481.55	268.46
Bin 5: [-0.87, -0.76]	137.00	73.53	54.78	685.67	439.04
Bin 6: [-0.76, -0.65]	202.00	161.44	10.19	961.69	600.12
Bin 7: [-0.65, -0.54]	284.00	351.65	13.01	1320.66	813.74
Bin 8: [-0.54, -0.44]	402.00	752.97	163.59	1762.05	1049.76
Bin 9: [-0.44, -0.33]	694.00	1554.91	476.66	2262.56	1087.44
Bin 10: [-0.33, -0.22]	1864.00	2983.47	420.05	2766.48	294.41
Bin 11: [-0.22, -0.11]	4447.00	4993.91	59.89	3187.16	498.00
Bin 12: [-0.11, 0.00]	5248.00	6714.82	320.42	3429.49	964.28
Bin 13: [0.00, 0.11]	5483.00	6792.32	252.39	3429.49	1229.61
Bin 14: [0.11, 0.22]	5130.00	5154.10	0.11	3187.16	1184.32
Bin 15: [0.22, 0.33]	1701.00	3121.98	646.76	2766.48	410.36
Bin 16: [0.33, 0.44]	688.00	1640.00	552.63	2262.56	1095.77
Bin 17: [0.44, 0.54]	407.00	797.39	191.13	1762.05	1042.06
Bin 18: [0.54, 0.65]	281.00	373.11	22.74	1320.66	818.45
Bin 19: [0.65, 0.76]	195.00	171.45	3.24	961.69	611.23
Bin 20: [0.76, 0.87]	135.00	78.12	41.41	685.67	442.25
Bin 21: [0.87, 0.98]	129.00	35.46	246.70	481.55	258.11
Bin 22: [0.98, 1.09]	82.00	16.07	270.48	334.62	190.72
Bin 23: [1.09, 1.20]	69.00	7.28	523.57	230.81	113.44
Bin 24: [1.20, 9.25]	713.00	6.02	83080.64	497.76	93.07

Tab. 6: Observed and estimated frequencies in a dataset comprising 39,357 samples (data from Eugene).

Bin edges	O_i	Mazumdar et al. fit		MLE fit	
		E_i	$(O_i - E_i)^2 / E_i$	E_i	$(O_i - E_i)^2 / E_i$
Bin 1: [-13.11, -1.23]	975.00	13.50	69117.35	487.74	484.42
Bin 2: [-1.23, -1.08]	192.00	23.49	1258.59	347.79	69.69
Bin 3: [-1.08, -0.93]	290.00	64.24	762.30	586.71	150.24
Bin 4: [-0.93, -0.77]	400.00	175.00	288.10	973.09	334.14
Bin 5: [-0.77, -0.62]	551.00	471.76	13.38	1569.28	676.79
Bin 6: [-0.62, -0.46]	818.00	1236.57	140.38	2419.80	1063.72
Bin 7: [-0.46, -0.31]	1720.00	3016.08	556.43	3484.88	829.64
Bin 8: [-0.31, -0.15]	6127.00	6223.22	1.50	4554.94	532.71
Bin 9: [-0.15, 0.00]	9643.00	9333.77	10.22	5254.25	3440.17

Bin 10: [0.00, 0.15]	9000.00	8953.56	0.24	5254.25	2654.17
Bin 11: [0.15, 0.31]	4884.00	5575.13	85.26	4554.94	23.76
Bin 12: [0.31, 0.46]	1600.00	2597.25	388.81	3484.88	955.36
Bin 13: [0.46, 0.62]	824.00	1045.96	46.63	2419.80	1075.71
Bin 14: [0.62, 0.77]	524.00	396.20	41.26	1569.28	689.38
Bin 15: [0.77, 0.93]	370.00	146.57	349.20	973.09	375.76
Bin 16: [0.93, 1.08]	246.00	53.75	701.23	586.71	197.11
Bin 17: [1.08, 1.23]	208.00	19.65	1760.46	347.79	56.32
Bin 18: [1.23, 13.11]	985.00	11.29	83911.36	487.74	507.93

Tab. 7: Pearson's $\chi 2$ test results.

Site Name	Mazumdar et al. fit			MLE fit		
	T	ν	T_C	T	ν	T_C
Cocoa	195476.14	21	32.67	13937.70	22	33.92
Eugene	158819.32	15	25.00	14464.41	16	26.30

By comparing the T values obtained for both models and both sites with their respective critical values, the null hypothesis is rejected at a 95% confidence level. Thus, it becomes evident, under the perspective of Pearson's χ^2 test, that power fluctuations in PV systems cannot be adequately modeled by a logistic distribution, at least for the scale, sampling time, and locations investigated. This behavior is consistent with the results previously discussed in the CDF, quantile and histogram analyses, reinforcing that although the logistic distribution can adequately represent small fluctuations, it fails to capture the statistical variability associated with higher-magnitude events.

5. Conclusion

This study presented an evaluation of the methodology proposed by Mazumdar et al. (2014) for characterizing power fluctuations in photovoltaic systems by means of a logistic distribution. In addition to employing the method proposed in the original paper for parameter estimation, MLE was also used to obtain a more robust parameter fit, thus assessing the characterization approach under its best possible conditions. The results demonstrate that it is possible to state, with a 95% confidence level, that the logistic distribution is not capable of adequately modeling the phenomenon under study, at least within the contexts of scale, sampling interval, and locations investigated. The largest discrepancies between the theoretical and experimental distributions are observed in the tail regions, indicating that the logistic model tends to underestimate the occurrence of extreme events.

Another contribution of this work lies in identifying and discussing potential limitations in the metrics used to assess the fits. Based on the findings presented, it is recommended that future studies in this field incorporate, within their methodologies, careful histogram comparisons (on a semilogarithmic scale), broad quantile comparisons, and, most importantly, hypothesis testing through Pearson's χ^2 test. In this context, subsequent research may pursue a comprehensive comparison among the different characterization methods available in the literature, taking as reference the evaluation guidelines established in this paper.

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